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NUMERICAL ANALYSIS OF FLOW AND PRESSURE FIELDS IN AN IDEALIZED SPIRAL-GROOVED PUMPING SEAL

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16. Abstract

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SUMMARY

A computer program is presented for finding the flow and pressure fields in a spiral-grooved pumping seal model for the limiting case of zero clearance. The governing nonlinear partial differential equations are solved numerically using the method of finite differences and over-relaxation. The program is written in FORTRAN IV and is completely described including the program listing, flow charts, and sample problem. The computer program calculates the net volume flow rate, velocity profiles, and pressure distributions for specific axial pressure gradients, Reynolds numbers, and aspect ratios. Other biharmonic problems may be solved using this program.

INTRODUCTION

In a companion paper (ref. 1) an analysis is given for the flow and pressure fields in a spiral-groove pumping seal model for the limiting case of zero clearance. A spiral-groove face seal (ref. 1) is a member of a general class of pressure generation devices that are characterized by two surfaces moving relative to each other with very small film thicknesses and with one or both surfaces grooved. Several geometric forms are found; for example, the cylindrical form (viscoseal), the herringbone groove bearing, and the conical and spherical bearing forms. Variations and combinations of these are also found. The numerical solutions of the exact governing equations using the numerical analysis and computer program presented herein are compared with classical models of the groove axial flow which neglect the coupling of the groove cross flow. The solutions presented in reference 1 included the following results: The cross flow shifts the pumping flow toward the land leading edge. Conditions under which the classical models give good approximations for the relation between axial pressure gradient and net volume flow are shown to depend on the Reynolds number and aspect ratio. The groove cross-

section static pressure is nearly constant except near the moving surface region. A low pressure region suggests the possibility of degassing and cavitation; a high-pressure region near the land leading edge results in a lift force acting on the moving surface.

The objective of this report is to present a method of solution and to present a computer program for numerical solutions for the flow and pressure fields in a spiral-grooved pumping seal model whose analysis is given in reference 1. Also, other physical problems that can be solved by the computer program are discussed. The computer program is written in FORTRAN IV for the Lewis Research Center IBM 7094II/7044 direct-couple system.

NUMERICAL ANALYSIS

Seal Model and Equations

As described in reference 1, the spiral-groove pumping seal model is a stationary rectangular cross-section groove with a wall (upper plate) moving at an oblique angle to the groove edges as illustrated in figure 1. In addition, a pressure gradient is imposed in the groove axial direction. A rectilinear Cartesian coordinate system is used. The flow is fully developed in the groove axial direction (z^*-direction); that is, end effects are neglected in the z^*-direction. The flow studied is for a homogeneous, incompressible Newtonian fluid under steady laminar flow conditions.

In reference 1 the flow field variables and resulting equations were nondimensionalized. (All symbols are defined in appendix A including dimensionless scaling values.)

In order to facilitate numerical analysis, the flow field equations across the groove (x^*-y^*-plane) are expressed in terms of a cross flow stream function $\psi^*(x^*, y^*)$ and the vorticity $\zeta^*(x^*, y^*)$. The derivatives of the stream function are related to the groove cross flow velocity components u^* and v^* such that

$$\left.\begin{aligned} u^* &= \frac{\partial \psi^*}{\partial y^*} \\ v^* &= -\frac{\partial \psi^*}{\partial x^*} \end{aligned}\right\} \tag{1}$$

In figure 2, the groove cross flow plane and the stream function direction are shown. Since the flow is fully developed in the z^*-direction (groove axial direction), $\partial w^*/\partial z^* \equiv 0$; thus, the use of the stream function automatically satisfies the dimensionless incompressible continuity equation

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \cot^2\alpha \, \frac{\partial w^*}{\partial z^*} = 0 \qquad (2)$$

The component of vorticity in the groove axial direction (z*-direction) is

$$\zeta^* = \lambda \frac{\partial v^*}{\partial x^*} - \frac{1}{\lambda}\frac{\partial u^*}{\partial y^*} = -\left(\lambda \frac{\partial^2 \psi^*}{\partial x^{*2}} + \frac{1}{\lambda}\frac{\partial^2 \psi^*}{\partial y^{*2}}\right) \qquad (3)$$

Two important dimensionless parameters were found from nondimensionalizing the governing equations:

$$\mathrm{Re} = \frac{bU \sin\alpha}{\nu}$$

and

$$\lambda = d/b$$

Groove Cross Flow Plane (x*-y* plane)

For the stated restrictions the appropriate flow field equation in the groove cross flow plane is the two-dimensional vorticity transport equation (see ref. 1) which reduces to

$$\frac{\partial^2 \zeta^*}{\partial x^{*2}} + \frac{1}{\lambda^2}\frac{\partial^2 \zeta^*}{\partial y^{*2}} = \mathrm{Re}\left(\frac{\partial \psi^*}{\partial y^*}\frac{\partial \zeta^*}{\partial x^*} - \frac{\partial \psi^*}{\partial x^*}\frac{\partial \zeta^*}{\partial y^*}\right) \qquad (4)$$

Groove Axial Flow Direction (z*-direction)

For fully developed flow in the z*-direction, the Navier-Stokes equation is

$$\frac{\partial \psi^*}{\partial y^*}\frac{\partial w^*}{\partial x^*} - \frac{\partial \psi^*}{\partial x^*}\frac{\partial w^*}{\partial y^*} = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w^*}{\partial x^{*2}} + \frac{1}{\lambda^2} + \frac{\partial^2 w^*}{\partial y^{*2}}\right) \qquad (5)$$

where

$$\frac{\partial P^*}{\partial z^*} = C_1 = \text{Constant}$$

Hence,

$$P^* = C_1 z^* + C_2(x^*, y^*) \tag{6}$$

The boundary conditions will now be stated: For convenience the stream function is chosen to be zero on the walls, hence,

$$\psi^*(x^*, 0) = 0 \qquad \psi^*(x^*, 1) = 0 \qquad \psi^*(0, y^*) = 0 \qquad \psi^*(1, y^*) = 0 \tag{7}$$

The fluid velocity no-slip and impermeability condition on the walls expressed in stream function form is

$$\frac{\partial \psi^*}{\partial y^*}(x^*, 0) = 0 \qquad \frac{\partial \psi^*}{\partial y^*}(x^*, 1) = 1 \qquad \frac{\partial \psi^*}{\partial x^*}(0, y^*) = 0 \qquad \frac{\partial \psi^*}{\partial x^*}(1, y^*) = 0 \tag{8}$$

And the no-slip condition for the groove axial direction velocity is

$$w^*(x^*, 0) = 0 \qquad w^*(x^*, 1) = -1 \qquad w^*(0, y^*) = 0 \qquad w^*(1, y^*) = 0 \tag{9}$$

Once the flow field is found in the x^*-y^* plane, the static pressure field can be found from the dimensionless Navier Stokes equations modified in the following way. using the dimensionless stream function and vorticity:

For $Re > 1$ (ref. 1):

$$\frac{\partial P^*}{\partial x^*} = -\left(\frac{\partial \psi^*}{\partial y^*}\,\frac{\partial^2 \psi^*}{\partial y^*\,\partial x^*} + \lambda^2\,\frac{\partial \psi^*}{\partial x^*}\,\frac{\partial^2 \psi^*}{\partial x^{*2}}\right) - \frac{1}{Re}\,\frac{1}{\lambda}\,\frac{\partial \zeta^*}{\partial y^*} - \lambda\zeta^*\,\frac{\partial \psi^*}{\partial x^*} \tag{10}$$

$$\frac{\partial P^*}{\partial y^*} = -\left(\frac{\partial \psi^*}{\partial y^*}\,\frac{\partial^2 \psi^*}{\partial y^{*2}} + \lambda^2\,\frac{\partial \psi^*}{\partial x^*}\,\frac{\partial^2 \psi^*}{\partial x^*\,\partial y^*}\right) + \frac{1}{Re}\,\lambda\,\frac{\partial \zeta^*}{\partial x^*} - \lambda\zeta^*\,\frac{\partial \psi^*}{\partial y^*} \tag{11}$$

For $0 < Re \leq 1$ (as stated in ref. 2, the pressure has to be rescaled for small values of Reynolds number):

$$\frac{1}{Re}\,\frac{\partial P^{*\prime}}{\partial x^*} = -\left(\frac{\partial \psi^*}{\partial y^*}\,\frac{\partial^2 \psi^*}{\partial y^*\,\partial x^*} + \lambda^2\,\frac{\partial \psi^*}{\partial x^*}\,\frac{\partial^2 \psi^*}{\partial x^{*2}}\right) - \frac{1}{Re}\,\frac{1}{\lambda}\,\frac{\partial \zeta^*}{\partial y^*} - \lambda\zeta^*\,\frac{\partial \psi^*}{\partial x^*} \tag{12}$$

$$\frac{1}{Re}\frac{\partial P^{*\prime}}{\partial y^*} = -\left(\frac{\partial \psi^*}{\partial y^*}\frac{\partial^2 \psi^*}{\partial y^{*2}} + \lambda^2 \frac{\partial \psi^*}{\partial x^*}\frac{\partial^2 \psi^*}{\partial x^* \partial y^*}\right) + \frac{1}{Re}\lambda \frac{\partial \zeta^*}{\partial x^*} - \lambda\zeta^* \frac{\partial \psi^*}{\partial y^*} \tag{13}$$

where $P^{*\prime} = ReP^*$.

In reference 2 results of the integration of the pressure distribution on the moving wall surface which yields a net lift force are shown. This net lift force per axial length is found from

$$\frac{F^*}{\text{Axial length}} = \int_0^1 P^* \, dx^* \tag{14}$$

In reference 2 it was further stated that if degassing occurred at the trailing edge (this region would be at ambient pressure) then a net lift force would occur only along the leading edge interval. The leading edge interval is defined as the range of x^* values from the point $x^*_{P^*>0}$ (where P^* is always greater than 0) to the point $x^* = 1$.

$$\frac{\text{Leading-edge force}}{\text{Axial length}} = \int_{x^*_{P^*>0}}^1 P^* \, dx^* \tag{15}$$

Outline of Solution

Due to the geometrical configuration and the nonlinearity of the flow field equation (4), analytical solutions are extremely difficult to obtain; however, equations (3), (4), (10), and (11) can be solved numerically.

The basic nondimensional flow field equations (3) and (4) are solved for the stream function ψ^* and vorticity ζ^* distributions using finite difference techniques and successive overrelaxation, similar to that used by Lieberstein (ref. 3). Once ψ^* and ζ^* are known, the normalized pressure field is calculated from equations (10) and (11), using a finite difference scheme suggested by Burggraf (ref. 4). From the results of equations (3) and (4) the dimensionless z^*-directional flow w^* and net volume flow Q_z^* can be calculated for specified values of the constant groove axial pressure gradient $\partial P^*/\partial z^*$. Equation (5) is solved by the method of finite differences for the w^*-field and Q_z^* is calculated from

$$Q_z^* = \int_0^1 \int_0^1 w^* \, dy^* \, dx^* \tag{16}$$

Finite Difference Method

A grid of mesh points (i, j) is constructed over the positive x^*-y^* plane (fig. 3) with i increasing for decreasing y^* values and j increasing for increasing x^* values. With this mesh, equations (3) to (5) can be developed into appropriate difference forms for solution on a digital computer. Central differencing techniques were used in the equations whenever possible; however, forward or backward differences were sometimes used especially at the walls. The FORTRAN IV computer programs are described in appendix C. The first program for the numerical solution of an idealized spiral-grooved pumping seal model calculates the vorticity and stream function, the u^*- and v^*-velocity profiles, the normalized pressure field, and the net lift forces. The second program calculates the w^*-velocity profile and dimensionless net volume flow rate along the groove axis.

The initial distribution for the stream function is assumed as a linear function with $\psi^* = 0$ at the walls and $\psi^* = -0.1$ at the center of the rectangular groove. For the case $\lambda = 1$, square groove, the stream function contours are squares of constant value. Various other distributions including ψ^* equal to a constant for all interior points were examined but were found to be less efficient in that more iterations were required to obtain convergence. The vorticity initial distribution is calculated at all interior points using equation (3) which in difference form becomes

$$\zeta^*_{i,j} = -\frac{\lambda}{\Delta x^{*2}}\left(\psi^*_{i,j+1} - 2\psi^*_{i,j} + \psi^*_{i,j-1}\right) - \frac{1}{\lambda \Delta y^{*2}}\left(\psi^*_{i-1,j} - 2\psi^*_{i,j} + \psi^*_{i+1,j}\right) \tag{17}$$

At the boundaries the initial vorticity values are calculated from equations (B3) and (B4). In finite difference notation these equations in dimensionless form, become

(1) Lower stationary wall

$$\zeta^*_{imax,j} = \frac{2}{\lambda \Delta y^{*2}}\left(\psi^*_{imax,j} - \psi^*_{imax-1,j}\right) \tag{18}$$

(2) Left stationary wall

$$\zeta^*_{i,1} = \frac{2\lambda}{\Delta x^{*2}}\left(\psi^*_{i,1} - \psi^*_{i,2}\right) \tag{19}$$

(3) Right stationary wall

$$\zeta^*_{i,jmax} = \frac{2\lambda}{\Delta x^{*2}} \left(\psi^*_{i,jmax} - \psi^*_{i,jmax-1}\right) \tag{20}$$

(4) Upper moving wall

$$\zeta^*_{1,j} = \frac{2}{\lambda \Delta y^{*2}} \left(\psi^*_{1,j} - \psi^*_{2,j} - \Delta y^*\right) \tag{21}$$

The solutions to equations (3) and (4) are calculated in an iterative routine in which the stream function or vorticity field is scanned once before entering the other field. In the literature, other authors (e.g., ref. 5), who have solved similar boundary value problems, have scanned each field a various number of times (from two to 50) before entering the other field. The present authors feel that this only overcorrects the values in that particular field, especially when a relaxation factor is used in the calculations. In the iterative process equation (4), which in difference form is

$$\frac{1}{\Delta x^{*2}} \left(\zeta^*_{i,j+1} - 2\zeta^*_{i,j} + \zeta^*_{i,j-1}\right) + \frac{1}{\lambda^2 \Delta y^{*2}} \left(\zeta^*_{i-1,j} - 2\zeta^*_{i,j} + \zeta^*_{i+1,j}\right) = \frac{Re}{4\Delta x^* \Delta y^*} \Bigg[\left(\psi^*_{i-1,j} - \psi^*_{i+1,j}\right)\left(\zeta^*_{i,j+1} - \zeta^*_{i,j-1}\right) - \left(\psi^*_{i\ j+1} - \psi^*_{i,j-1}\right)\left(\zeta^*_{i-1,j} - \zeta^*_{i+1,j}\right)\Bigg] \tag{22}$$

serves as the basis for the vorticity calculations at the interior points; equations (18) to (21) are used for the vorticity boundary values. The values of the stream function at the interior mesh points in the iterative process are calculated from equation (17), while the boundary values are fixed at $\psi^* = 0$.

The successive overrelaxation technique used in this analysis is based on a paper by Lieberstein (ref. 3). The basic iterative relaxation equation is

$$y_i^{n+1} = y_i^n - \omega \frac{f(y_1, y_2, \ldots, y_k)}{f'(y_1, y_2, \ldots, y_k)} \tag{23}$$

where y_i^n and y_i^{n+1} are either $\psi^*_{i,j}$ or $\zeta^*_{i,j}$ at the n or n+1 iteration; $f(y_1, y_2, \ldots y_k)$ is either equation (17) or (22); $f'(y_1, y_2, \ldots, y_k)$ is the combined coefficients of $\psi^*_{i,j}$ from equation (17) or the combined coefficients of $\zeta^*_{i,j}$ from equa-

tion (22); and ω is the relaxation factor. In evaluating $f(y_1, y_2, \ldots, y_k)$ and $f'(y_1, y_2, \ldots, y_k)$ the most recent available values for the y's are used. Substituting the appropriate terms into equation (23) yields the following equations for the vorticity and stream function as coded in the computer program:

$$\zeta^{*n+1}_{i,j} = (1-\omega)\zeta^{*n}_{i,j} - K_1\left\{\left(\frac{\zeta^{*n}_{i,j+1} + \zeta^{*n+1}_{i,j-1}}{\Delta x^{*2}} + \frac{\zeta^{*n+1}_{i-1,j} + \zeta^{*n}_{i+1,j}}{\lambda^2\,\Delta y^{*2}}\right) - \frac{Re}{4\Delta x^*\,\Delta y^*}\right.$$

$$\times\left[\left(\psi^{*n}_{i,j+1} - \psi^{*n}_{i,j-1}\right)\left(\zeta^{*n+1}_{i-1,j} - \zeta^{*n}_{i+1,j}\right)\right.$$

$$\left.\left. - \left(\psi^{*n}_{i-1,j} - \psi^{*n}_{i+1,j}\right)\left(\zeta^{*n}_{i,j+1} - \zeta^{*n+1}_{i,j-1}\right)\right]\right\} \qquad (24)$$

$$\psi^{*n+1}_{i,j} = (1-\omega)\psi^{*n}_{i,j} - \frac{K_1}{\lambda}\left[\frac{\lambda}{\Delta x^{*2}}\left(\psi^{*n}_{i,j+1} + \psi^{*n+1}_{i,j-1}\right)\right.$$

$$\left. + \frac{1}{\lambda\,\Delta y^{*2}}\left(\psi^{*n+1}_{i-1,j} + \psi^{*n}_{i+1,j}\right) + \zeta^{*n+1}_{i,j}\right] \qquad (25)$$

where

$$K_1 = \frac{\omega}{-2\left(\dfrac{1}{\Delta x^{*2}} + \dfrac{1}{\lambda^2\,\Delta y^{*2}}\right)}$$

Values of ω between 0 and 2 are used as relaxation factors, but the optimum value for most rapid convergence depends on the aspect ratio and mesh size. For $\omega = 1$, this iterative process reduces to the Liebmann iterated forms as used by Mills (ref. 5).

Once the stream function field has been found the u^* and v^* velocity components in the x^*- and y^*-directions, respectively, can be calculated from equation (1).

The stream function boundary conditions (eqs. (7) to (9)) specified in terms of velocity boundary conditions are

$$u^*(0, y^*) = u^*(1, y^*) = u^*(x^*, 0) = 0, \quad u^*(x^*, 1) = 1$$

$$v^*(0, y^*) = v^*(1, y^*) = v^*(x^*, 0) = v^*(x^*, 1) = 0 \tag{26}$$

In central difference notation these equations become

$$\left.\begin{aligned} u^*_{i,j} &= \frac{\psi^*_{i-1,j} - \psi^*_{i+1,j}}{2\,\Delta y^*} \\ &\text{and} \\ v^*_{i,j} &= -\frac{\psi^*_{i,j+1} - \psi^*_{i,j-1}}{2\,\Delta x^*} \end{aligned}\right\} \tag{27}$$

These equations are used in this form to calculate the velocities at all interior mesh points.

The static pressure field is obtained by a point-to-point integration of equations (10) and (11) over a single mesh width. At each mesh point the values obtained from the two equations are averaged to find one pressure for each point. Substituting equation (1) into equations (10) and (11) and rearranging terms yields the following forms from which the static pressure is calculated:

$$\frac{\partial P^*}{\partial x^*} = -\frac{1}{Re}\frac{1}{\lambda}\frac{\partial \zeta^*}{\partial y^*} + \lambda\zeta^* v^* - u^* \frac{\partial^2 \psi^*}{\partial y^*\,\partial x^*} + \lambda^2 v^* \frac{\partial^2 \psi^*}{\partial x^{*2}} \tag{28}$$

$$\frac{\partial P^*}{\partial y^*} = \frac{1}{Re}\lambda\frac{\partial \zeta^*}{\partial x^*} - \lambda\zeta^* u^* - u^* \frac{\partial^2 \psi^*}{\partial y^{*2}} + \lambda^2 v^* \frac{\partial^2 \psi^*}{\partial x^*\,\partial y^*} \tag{29}$$

For the case $0 < Re \leq 1$, the corresponding static pressure ($P^{*\prime}$) equations are obtained by multiplying the right side of equations (28) and (29) by Re.

In the finite-difference representation of these equations the pressure partial derivatives are approximated using forward or backward differences and the values of the pressure at adjacent mesh points. Each of the terms on the right side is calculated at the same pair of adjacent points and then corresponding terms are averaged to give one approximation for each term. For example, in equation (28), the pressure term is approxi-

mated using points (i, j) and (i, j+1) on the grid. Each term on the right side is then evaluated at (i, j) and at (i, j+1); the value of a term from (i, j) is averaged with the value from (i, j+1) to give one approximation in the finite-difference representation.

To adjust for the discontinuity in the vorticity at the junctions of the moving wall with the stationary walls the following scheme was adopted for these two points:

(1) If the term being defined is a partial derivative, the value of the vorticity is chosen in the same direction as the derivative; for example, the values for $\partial\zeta^*/\partial x^*$ are calculated from equation (21), and the values for $\partial\zeta^*/\partial y^*$ are calculated from equations (19) or (20).

(2) If the term being defined is the vorticity itself, the value chosen is dependent on the direction of the pressure derivative term; for example, the value of the vorticity used in equation (28) is calculated from equation (21), and the value in equation (29) is calculated from either equation (19) or (20). The finite-difference equations representing equations (28) and (29) at the interior mesh-points, along the walls, and at the corners are thus given by

(1) Interior points

$$
\begin{aligned}
\frac{P^*_{i,j} - P^*_{i,j-1}}{\Delta x^*} = &-\frac{1}{Re\lambda}\,\frac{1}{2}\left(\frac{\zeta^*_{i-1,j} - \zeta^*_{i+1,j}}{2\,\Delta y^*} + \frac{\zeta^*_{i-1,j-1} - \zeta^*_{i+1,j-1}}{2\,\Delta y^*}\right) + \frac{\lambda}{2}\left(\zeta^*_{i,j}v^*_{i,j} + \zeta^*_{i,j-1}v^*_{i,j-1}\right) \\
&- \frac{1}{2}\left[u^*_{i,j}\left(\frac{\psi^*_{i-1,j+1} - \psi^*_{i+1,j-1} + \psi^*_{i+1,j-1} - \psi^*_{i+1,j+1}}{4\,\Delta x^*\,\Delta y^*}\right)\right. \\
&\left. + u^*_{i,j-1}\left(\frac{\psi^*_{i-1,j} - \psi^*_{i-1,j-2} + \psi^*_{i+1,j-2} - \psi^*_{i+1,j}}{4\,\Delta x^*\,\Delta y^*}\right)\right] \\
&+ \frac{\lambda^2}{2}\left[v^*_{i,j}\left(\frac{\psi^*_{i,j+1} - 2\psi^*_{i,j} + \psi^*_{i,j-1}}{\Delta x^{*2}}\right)\right. \\
&\left. + v^*_{i,j-1}\left(\frac{\psi^*_{i,j} - 2\psi^*_{i,j-1} + \psi^*_{i,j-2}}{\Delta x^{*2}}\right)\right] \qquad (30)
\end{aligned}
$$

$$\frac{P^*_{i,j} - P^*_{i+1,j}}{\Delta y^*} = \frac{\lambda}{Re}\,\frac{1}{2}\left(\frac{\zeta^*_{i,j+1} - \zeta^*_{i,j-1}}{2\,\Delta x^*} + \frac{\zeta^*_{i+1,j+1} - \zeta^*_{i+1,j-1}}{2\,\Delta x^*}\right) - \frac{\lambda}{2}\left(\zeta^*_{i,j}u^*_{i,j} + \zeta^*_{i+1,j}u^*_{i+1,j}\right)$$

$$- \frac{1}{2}\left[u^*_{i,j}\left(\frac{\psi^*_{i-1,j} - 2\psi^*_{i,j} + \psi^*_{i+1,j}}{\Delta y^{*2}}\right)\right.$$

$$\left. + u^*_{i+1,j}\left(\frac{\psi^*_{i,j} - 2\psi^*_{i+1,j} + \psi^*_{i+2,j}}{\Delta y^{*2}}\right)\right]$$

$$+ \frac{\lambda^2}{2}\left[v^*_{i,j}\left(\frac{\psi^*_{i-1,j+1} - \psi^*_{i-1,j-1} + \psi^*_{i+1,j-1} - \psi^*_{i+1,j+1}}{4\,\Delta x^*\,\Delta y^*}\right)\right.$$

$$\left. + v^*_{i+1,j}\left(\frac{\psi^*_{i,j+1} - \psi^*_{i,j-1} + \psi^*_{i+2,j-1} - \psi^*_{i+2,j+1}}{4\,\Delta x^*\,\Delta y^*}\right)\right] \qquad (31)$$

(2) Boundary points

In equations (32) through (47) which follow, the zero-valued terms due to the boundary conditions of the spiral-grooved pumping seal have been omitted.

(a) Stationary wall ($x^* = 0,\ 0 < y^* < 1$)

$$\frac{P^*_{i,2} - P^*_{i,1}}{\Delta x^*} = -\frac{1}{Re\lambda}\,\frac{1}{2}\left(\frac{\zeta^*_{i-1,1} - \zeta^*_{i+1,1}}{2\,\Delta y^*} + \frac{\zeta^*_{i-1,2} - \zeta^*_{i+1,2}}{2\,\Delta y^*}\right) + \frac{\lambda}{2}\,\zeta^*_{i,2}v^*_{i,2} - \frac{u^*_{i,2}}{2}$$

$$\times\left(\frac{\psi^*_{i-1,3} - \psi^*_{i-1,2} + \psi^*_{i+1,2} - \psi^*_{i+1,3}}{2\,\Delta x^*\,\Delta y^*}\right) + \lambda^2\,\frac{v^*_{i,2}}{2}\left(\frac{\psi^*_{i,4} - 2\psi^*_{i,3} + \psi^*_{i,2}}{\Delta x^{*2}}\right) \qquad (32)$$

$$\frac{P^*_{i,1} - P^*_{i+1,1}}{\Delta y^*} = \frac{\lambda}{Re}\,\frac{1}{2}\left(\frac{\zeta^*_{i,2} - \zeta^*_{i,1}}{\Delta x^*} + \frac{\zeta^*_{i+1,2} - \zeta^*_{i+1,1}}{\Delta x^*}\right) \tag{33}$$

(b) Stationary wall ($x^* = 1,\ 0 < y^* < 1$)

$$\frac{P^*_{i,jmax} - P^*_{i,jmax-1}}{\Delta x^*} = -\frac{1}{Re\lambda}\,\frac{1}{2}\left(\frac{\zeta^*_{i-1,jmax} - \zeta^*_{i+1,jmax}}{2\,\Delta y^*} + \frac{\zeta^*_{i-1,jmax-1} + \zeta^*_{i+1,jmax-1}}{2\,\Delta y^*}\right)$$

$$+ \frac{\lambda}{2}\,\zeta^*_{i,jmax-1} v^*_{i,jmax-1} - \frac{u^*_{i,jmax-1}}{2}$$

$$\times \left(\frac{\psi^*_{i-1,jmax-1} - \psi^*_{i-1,jmax-2} + \psi^*_{i+1,jmax-2} - \psi^*_{i+1,jmax-1}}{2\,\Delta x^*\,\Delta y^*}\right)$$

$$+ \lambda^2\,\frac{v^*_{i,jmax-1}}{2}\left(\frac{\psi^*_{i,jmax-1} - 2\psi^*_{i,jmax-2} + \psi^*_{i,jmax-3}}{\Delta x^{*2}}\right) \tag{34}$$

$$\frac{P^*_{i,jmax} - P^*_{i+1,jmax}}{\Delta y^*} = \frac{\lambda}{Re}\,\frac{1}{2}\left(\frac{\zeta^*_{i,jmax} - \zeta^*_{i,jmax-1}}{\Delta x^*} + \frac{\zeta^*_{i+1,jmax} - \zeta^*_{i+1,jmax-1}}{\Delta x^*}\right) \tag{35}$$

(c) Stationary wall ($y^* = 0,\ 0 < x^* < 1$)

$$\frac{P^*_{imax,j} - P^*_{imax,j-1}}{\Delta x^*} = -\frac{1}{Re\lambda}\,\frac{1}{2}$$

$$\times\left(\frac{\zeta^*_{imax-1,j} - \zeta^*_{imax,j}}{\Delta y^*} + \frac{\zeta^*_{imax-1,j-1} - \zeta^*_{imax,j-1}}{\Delta y^*}\right) \tag{36}$$

$$\frac{P^*_{i,max-1,j} - P^*_{imax,j}}{\Delta y^*} = \frac{\lambda}{Re}\,\frac{1}{2}\left(\frac{\zeta^*_{imax,j+1} - \zeta^*_{imax,j-1}}{2\,\Delta x^*} + \frac{\zeta^*_{imax-1,j+1} - \zeta^*_{imax-1,j-1}}{2\,\Delta x^*}\right)$$

$$- \frac{\lambda}{2}\,\zeta^*_{imax-1,j}u^*_{imax-1,j} - \frac{u^*_{imax-1,j}}{2}$$

$$\times\left(\frac{\psi^*_{imax-3,j} - 2\psi^*_{imax-2,j} + \psi^*_{imax-1,j}}{\Delta y^{*2}}\right) + \lambda^2\,\frac{v^*_{imax-1,j}}{2}$$

$$\times\left(\frac{\psi^*_{imax-2,j+1} - \psi^*_{imax-2,j-1} + \psi^*_{imax-1,j-1} - \psi^*_{imax-1,j+1}}{2\,\Delta x^*\,\Delta y^*}\right) \tag{37}$$

(d) Moving wall ($y^* = 1,\ 0 < x^* < 1$)

$$\frac{P^*_{1,j} - P^*_{1,j-1}}{\Delta x^*} = -\frac{1}{Re\lambda}\,\frac{1}{2}\left(\frac{\zeta^*_{1,j} - \zeta^*_{2,j}}{\Delta y^*} + \frac{\zeta^*_{1,j-1} - \zeta^*_{2,j-1}}{\Delta y^*}\right)$$

$$- \frac{1}{2}\left(\frac{\psi^*_{1,j+1} - \psi^*_{1,j-1} + \psi^*_{2,j-1} - \psi^*_{2,j+1}}{2\,\Delta x^*\,\Delta y^*}\right.$$

$$\left. + \frac{\psi^*_{1,j} - \psi^*_{1,j-2} + \psi^*_{2,j-2} - \psi^*_{2,j}}{2\,\Delta x^*\,\Delta y^*}\right) \tag{38}$$

$$\frac{P^*_{1,j} - P^*_{2,j}}{\Delta y^*} = \frac{\lambda}{Re}\,\frac{1}{2}\left(\frac{\zeta^*_{1,j+1} - \zeta^*_{1,j-1}}{2\,\Delta x^*} + \frac{\zeta^*_{2,j+1} - \zeta^*_{2,j-1}}{2\,\Delta x^*}\right) - \frac{\lambda}{2}\left(\zeta^*_{1,j} + \zeta^*_{2,j} u^*_{2,j}\right)$$

$$- \frac{1}{2}\left[\left(\frac{\psi^*_{1,j} - 2\psi^*_{2,j} + \psi^*_{3,j}}{\Delta y^{*2}}\right) + u^*_{2,j}\left(\frac{\psi^*_{2,j} - 2\psi^*_{3,j} + \psi^*_{4,j}}{\Delta y^{*2}}\right)\right]$$

$$+ \lambda^2 \frac{v^*_{2,j}}{2}\left(\frac{\psi^*_{2,j+1} - \psi^*_{2,j-1} + \psi^*_{3,j-1} - \psi^*_{3,j+1}}{2\,\Delta x^*\,\Delta y^*}\right) \tag{39}$$

(e) Corner point ($x^* = 0$, $y^* = 1$)

$$\frac{P^*_{1,2} - P^*_{1,1}}{\Delta x^*} = -\frac{1}{Re\lambda}\,\frac{1}{2}\left(\frac{-\zeta^*_{2,1}}{\Delta y^*} + \frac{\zeta^*_{1,2} - \zeta^*_{2,2}}{\Delta y^*}\right) - \frac{1}{2}\left(\frac{\psi^*_{1,2} - \psi^*_{1,1} + \psi^*_{2,1} - \psi^*_{2,2}}{\Delta x^*\,\Delta y^*}\right.$$

$$\left. + \frac{\psi^*_{1,3} - \psi^*_{1,2} + \psi^*_{2,2} - \psi^*_{2,3}}{\Delta x^*\,\Delta y^*}\right) \tag{40}$$

$$\frac{P^*_{1,1} - P^*_{2,1}}{\Delta y^*} = \frac{\lambda}{Re}\,\frac{1}{2}\left(\frac{\zeta^*_{1,2} - \zeta^*_{1,1}}{\Delta x^*} + \frac{\zeta^*_{2,2} - \zeta^*_{2,1}}{\Delta x^*}\right) \tag{41}$$

(f) Corner point ($x^* = 1$, $y^* = 1$)

$$\frac{P^*_{1,jmax} - P^*_{1,jmax-1}}{\Delta x^*} = -\frac{1}{Re\lambda}\,\frac{1}{2}\left(\frac{-\zeta^*_{2,jmax}}{\Delta y^*} + \frac{\zeta^*_{1,jmax-1} - \zeta^*_{2,jmax-1}}{\Delta y^*}\right)$$

$$-\frac{1}{2}\left(\frac{\psi^*_{1,jmax} - \psi^*_{1,jmax-1} + \psi^*_{2,jmax-1} - \psi^*_{2,jmax}}{\Delta x^*\,\Delta y^*}\right.$$

$$\left. + \frac{\psi^*_{1,jmax-1} - \psi^*_{1,jmax-2} + \psi^*_{2,jmax-2} - \psi^*_{2,jmax-1}}{\Delta x^*\,\Delta y^*}\right) \tag{42}$$

$$\frac{P^*_{1,jmax} - P^*_{2,jmax}}{\Delta y^*} = \frac{\lambda}{Re}\,\frac{1}{2}\left(\frac{\zeta^*_{1,jmax} - \zeta^*_{1,jmax-1}}{\Delta x^*} + \frac{\zeta^*_{2,jmax} - \zeta^*_{2,jmax-1}}{\Delta x^*}\right) \tag{43}$$

(g) Corner point ($x^* = 0$, $y^* = 0$)

$$\frac{P^*_{imax,2} - P^*_{imax,1}}{\Delta x^*} = -\frac{1}{Re\lambda}\,\frac{1}{2}\left(\frac{\zeta^*_{imax-1,1} - \zeta^*_{imax,1}}{\Delta y^*} + \frac{\zeta^*_{imax-1,2} - \zeta^*_{imax,2}}{\Delta y^*}\right) \tag{44}$$

$$\frac{P^*_{imax-1,1} - P^*_{imax,1}}{\Delta y^*} = \frac{\lambda}{Re}\,\frac{1}{2}\left(\frac{\zeta^*_{imax,2} - \zeta^*_{imax,1}}{\Delta x^*} + \frac{\zeta^*_{imax-1,2} - \zeta^*_{imax-1,1}}{\Delta x^*}\right) \tag{45}$$

(h) Corner point ($x^* = 1$, $y^* = 0$)

$$\frac{P^*_{imax,jmax} - P^*_{imax,jmax-1}}{\Delta x^*} = -\frac{1}{Re\lambda}\,\frac{1}{2}\left(\frac{\zeta^*_{imax-1,jmax} - \zeta^*_{imax,jmax}}{\Delta y^*}\right.$$

$$\left. + \frac{\zeta^*_{imax-1,jmax-1} - \zeta^*_{imax,jmax-1}}{\Delta y^*}\right) \tag{46}$$

$$\frac{P^*_{imax-1,\,jmax} - P^*_{imax,\,jmax}}{\Delta y^*} = \frac{\lambda}{Re}\,\frac{1}{2}\left(\frac{\zeta^*_{imax,\,jmax} - \zeta^*_{imax,\,jmax-1}}{\Delta x^*} + \frac{\zeta^*_{imax-1,\,jmax} - \zeta^*_{imax-1,\,jmax-1}}{\Delta x^*}\right) \tag{47}$$

The initial distribution across the entire static pressure field is constant at $P^* = 1$. The program then iterates on the pressure field from top-to-bottom and left-to-right until convergence is achieved. The normalized static pressure field is calculated by subtracting from all the pressures in the field the value at one reference point specified as a program input. It is felt that this normalization method presents more useful results rather than the pressure ratios presented in references 1 and 2.

Both integrations in equations (14) and (15) in finding the net lift force per axial length and leading edge force were performed numerically using Simpson's rule.

In program two, the w*-velocity profile is calculated based on the stream function distribution calculated in program one. The stream function data are read in to program two, and the w*-field is initially equal to one, except for the boundary conditions presented with equation (5). If for a certain set of parameters, Re and λ, the w*-field is to be calculated for more than one value of $\partial P^*/\partial z^*$, the initial value of the w*-field for the succeeding $\partial P^*/\partial z^*$ value is set equal to the converged value from the preceeding $\partial P^*/\partial z^*$. Lieberstein's (ref. 3) successive overrelaxation technique is again used in the solution of equation (5) with the y_i's of equation (23) being equal to the latest available values of $w^*_{i,j}$. The $f(y_1, y_2, \ldots, y_k)$ is evaluated from the following equation which is the finite difference form of equation (5):

$$\left(\frac{\psi^*_{i-1,\,j} - \psi^*_{i+1,\,j}}{2\,\Delta y^*}\right)\left(\frac{w^*_{i,\,j+1} - w^*_{i,\,j-1}}{2\,\Delta x^*}\right) - \left(\frac{\psi^*_{i,\,j+1} - \psi^*_{i,\,j-1}}{2\,\Delta x^*}\right)\left(\frac{w^*_{i-1,\,j} - w^*_{i+1,\,j}}{2\,\Delta y^*}\right) + \frac{\partial P^*}{\partial z^*}$$
$$- \frac{1}{Re}\left[\left(\frac{w^*_{i,\,j+1} - 2w^*_{i,\,j} + w^*_{i,\,j-1}}{\Delta x^{*2}}\right) + \frac{1}{\lambda^2}\left(\frac{w^*_{i-1,\,j} - 2w^*_{i,\,j} + w^*_{i+1,\,j}}{\Delta y^{*2}}\right)\right] = 0 \tag{48}$$

And $f'(y_1, y_2, \ldots, y_n)$ is the combined coefficients of $w^*_{i,j}$ from the preceding equation.

Substituting the appropriate expressions for f and f' into equation (23) yields the z^*-directional flow equation as coded in program two:

$$w^{*n+1}_{i,j} = (1-\omega)w^{*n}_{i,j} + \text{Re}\, K_1 \left\{ \left(\frac{\psi^*_{i-1,j} - \psi^*_{i+1,j}}{2\,\Delta y^*} \right) \left(\frac{w^{*n}_{i,j+1} - w^{*n+1}_{i,j-1}}{2\,\Delta x^*} \right) \right.$$

$$- \left(\frac{\psi^*_{i,j+1} - \psi^*_{i,j-1}}{2\,\Delta x^*} \right) \left(\frac{w^{*n+1}_{i-1,j} - w^{*n}_{i+1,j}}{2\,\Delta y^*} \right)$$

$$\left. + \frac{\partial P^*}{\partial z^*} - \frac{1}{\text{Re}} \left[\left(\frac{w^{*n}_{i,j+1} + w^{*n+1}_{i,j-1}}{\Delta x^{*2}} \right) + \frac{1}{\lambda^2} \left(\frac{w^{*n+1}_{i-1,j} + w^{*n}_{i+1,j}}{\Delta y^{*2}} \right) \right] \right\} \tag{49}$$

where K_1 is defined in connection with (24) and (25). The range of the relaxation factor ω is between 0 and 2.0 with an optimum value of approximately 1.3 for $\lambda = 1$ and 29 mesh points in the x^*- and y^*-directions. Other combinations of λ and grid size have different optimum relaxation factors.

After the w^*-field has iterated to convergence within the prescribed error condition, the net volume flow is calculated from equation (16) using the method of mechanical cubature. A requirement of this method, which is based on Simpson's Rule of integration for two dimensions, is that there be an odd number of mesh points in both the x^*- and y^*-directions so that the proper weight factors will be applied in the cubature scheme. The double integral in equation (16) is thus approximated by the double summation as given by

$$Q^*_z = \frac{\Delta x^*\ \Delta y^*}{9} \sum_{i=2,4,6,\ldots}^{imax-1} \ \sum_{j=2,4,6,\ldots}^{jmax-1} 16 w^*_{i,j} + 4\left(w^*_{i-1,j} + w^*_{i+1,j} + w^*_{i,j-1} + w^*_{i,j+1}\right)$$

$$+ w^*_{i-1,j-1} + w^*_{i-1,j+1} + w^*_{i+1,j-1} + w^*_{i+1,j+1} \tag{50}$$

Convergence Remarks

A solution was considered to have converged when the following criteria were met:

1. The relative change in the values of the stream function, pressure distribution, or axial velocity distribution between two successive iterations was less than a prescribed maximum for all mesh points; for example,

$$\frac{f^{n+1} - f^{n}}{f^{n+1}} < \text{Prescribed maximum}$$

2. The values of the stream function and vorticity at any mesh point changed less than 1 percent when the number of mesh points was doubled in either direction.

In addition to the convergence criteria, the effect of round-off error on the results of reference 1 was checked by doubling the precision of the computing machine calculations.

Computer Program Formulation

As mentioned in the previous section, the equations in finite difference form are solved on a high-speed digital computer. A complete description of the program is presented in appendix C. The program listing is given in appendix D. Figures 4 to 12 present the computer program flow charts. Flow charts of the main program, stream function and vorticity, u^*- and v^*-velocities, pressure field, w^*-velocities, and net volume flow Q_z^* calculations are shown. A sample problem for the case where $\lambda = 1$, $Re = 100$, and $(\partial P^*)/(\partial z^*) = -0.115$ with its input and output is given in appendix E. (Plots of this case are in ref. 1.)

Program Use for Other Physical Problems

The computer program finds the stream function and vorticity in the groove cross-flow plane by solving equations (3) and (4), which are the following:

$$\nabla^2 \psi^* = -\zeta^* \tag{3}$$

$$\frac{\partial^2 \zeta^*}{\partial x^{*2}} + \frac{1}{\lambda^2} \frac{\partial^2 \zeta^*}{\partial y^{*2}} = Re\left(\frac{\partial \psi^*}{\partial y^*} \frac{\partial \zeta^*}{\partial x^*} - \frac{\partial \psi^*}{\partial x^*} \frac{\partial \zeta^*}{\partial y^*}\right) \tag{4}$$

These equations can be combined and placed in the following form (The aspect ratio λ can be eliminated by redefining the independent variables):

$$\nabla^4 \psi^* = \mathrm{Re} \frac{\partial(\psi^*, \nabla^2 \psi^*)}{\partial(x^*, y^*)} \tag{51}$$

The computer program can be used to solve many problems in mathematical physics that appear in this form or are reducible to this form. For example, the biharmonic equation (This is the creeping flow case discussed in ref. 1) $\nabla^4 \psi^* = 0$

Laplace's equation

$$\nabla^2 \psi^* = 0$$

Poisson's equation

$$\nabla^2 \psi^* = -\zeta^*$$

Of course, the proper transformation of the variables must be made to the form solved in the computer program. The proper boundary conditions must be specified.

The coding in the FORTRAN IV computer program contains all the boundary terms including the zero-valued terms omitted in equations (32) through (47) which were specifically written for the spiral-grooved pumping seal. The configuration must be rectangular.

Lewis Research Center,
National Aeronautics and Space Administration,
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126-15.

APPENDIX A

SYMBOLS

b	groove width
C	numerical coefficient
d	groove depth
F	net moving surface lift force
F^*	dimensionless lift force, $(F/b\rho U^2 \sin^2 \alpha) \times$ (total axial length)
K_1	numerical coefficient
n	iteration number
P	static pressure
$P^{*\prime}$	dimensionless pressure, $Pb/\mu U \sin \alpha$ (for $0 < Re \leq 1$)
P^*	dimensionless pressure, $P/\rho U^2 \sin^2 \alpha$ (for $Re > 1$)
$\frac{\partial P^*}{\partial z^*}$	dimensionless groove axial pressure gradient, constant
Q_z	net volume flow rate, groove axial direction
Q_z^*	dimensionless net volume flow rate, $Q_z/(dbU \cos \alpha)$
Re	Reynolds number, $(bU \sin \alpha)/\nu$
U	moving wall velocity
u	velocity in x-direction
u^*	dimensionless velocity, $u/(U \sin \alpha)$
v	velocity in y-direction
v^*	dimensionless velocity, $b/d\left[v/(U \sin \alpha)\right]$
w	velocity in z-direction
w^*	dimensionlesss velocity, $w/(U \cos \alpha)$
x	cross groove coordinate
x^*	dimensionless coordinate = x/b
Δx^*	mesh width in x^*-direction
y	groove depth direction coordinate
y^*	dimensionless coordinate = y/d

Δy^*	mesh width in y^*-direction
z	groove axial coordinate
z^*	dimensionless coordinate, $z/(b \tan \alpha)$
α	angle between moving wall direction and groove axis
λ	aspect ratio, equal to groove depth to groove width, d/b
μ	absolute viscosity of fluid
ν	kinematic viscosity of fluid
ψ	stream function, defined in x-y plane
ψ^*	dimensionless stream function, $\psi/(dU \sin \alpha)$
ζ	vorticity component in z-direction
ζ^*	dimensionless vorticity, $\zeta b/(U \sin \alpha)$
ω	relaxation factor
∇^2	Laplacian operator, $\left(\lambda \frac{\partial^2}{\partial x^{*2}} + \frac{1}{\lambda} \frac{\partial^2}{\partial y^{*2}}\right)$
∇^4	biharmonic operator, $\left(\lambda \frac{\partial^4}{\partial x^{*4}} + \frac{2}{\lambda} \frac{\partial^4}{\partial x^{*2}\, \partial y^{*2}} + \frac{\partial^4}{\lambda\, \partial y^{*4}}\right)$
$\frac{\partial(\psi^*, \nabla^2\psi^*)}{\partial(x^*, y^*)}$	Jacobian operator, $\left(\frac{\partial\psi^*}{\partial x^*} \frac{\partial\nabla^2\psi^*}{\partial y^*} - \frac{\partial\psi^*}{\partial y^*} \frac{\partial\nabla^2\psi^*}{\partial x^*}\right)$

Subscripts:

i	mesh point in y^*-direction
imax	maximum value of mesh point in y^*-direction ($y^* = 0$)
j	mesh point in x^*-direction
jmax	maximum value of mesh point in x^*-direction ($x^* = 1$)
0	boundary reference point

APPENDIX B

FIRST ORDER VORTICITY BOUNDARY VALUE APPROXIMATION

The boundary conditions for the stream function are known on the boundary because the normal and tangential velocities must satisfy the impermeability and no-slip conditions at the wall. The vorticity value on the boundary must be calculated and will vary from iteration to iteration until a specified convergence criterion is satisfied. The vorticity at the boundary can be found by the following first order approximation.

Consider the boundary along the x*-axis as shown in figure 13. The stream function expanded in a Taylor series about point 0 is

$$\psi_1^* = \psi_0^* + (\Delta y^*)\left(\frac{\partial \psi^*}{\partial y^*}\right)_0 + \frac{(\Delta y^*)^2}{2!}\left(\frac{\partial^2 \psi^*}{\partial y^{*2}}\right)_0 + \mathcal{O}\left[(\Delta y^*)\right]^3 \tag{B1}$$

Using equation (3) which relates the stream and vorticity functions yields

$$\lambda\left(\frac{\partial^2 \psi^*}{\partial x^{*2}}\right)_0 + \frac{1}{\lambda}\left(\frac{\partial^2 \psi^*}{\partial y^{*2}}\right)_0 = -\zeta_0^* \tag{B2}$$

Since $\psi^* = \psi^*(x^*)$ on the boundary shown in figure 13,

$$\left(\frac{\partial^2 \psi^*}{\partial x^{*2}}\right) = 0$$

Substituting equation (B2) into the truncated Taylor series expansion (B1) results in

$$\zeta_0^* = \frac{2}{\lambda(\Delta y^*)^2}\left[\psi_0^* - \psi_1^* + (\Delta y^*)\left(\frac{\partial \psi^*}{\partial y^*}\right)_0\right] \tag{B3}$$

In like manner if the wall were along the y*-axis, the wall vorticity equation is

$$\zeta_0^* = \frac{2\lambda}{(\Delta x^*)^2}\left[\psi_0^* - \psi_1^* + (\Delta x^*)\left(\frac{\partial \psi^*}{\partial x^*}\right)_0\right] \tag{B4}$$

The resulting vorticity boundary conditions are found by substituting the boundary conditions equations (7) and (8) into equations (B3) and (B4) and are for stationary walls:

$$\zeta_0^* = \frac{-2\psi_1^*}{\lambda(\Delta y^*)^2} \qquad \text{(horizontal)}$$

$$\zeta_0^* = \frac{-2\lambda\psi_1^*}{(\Delta x^*)^2} \qquad \text{(vertical)}$$

and for a moving wall

$$\zeta_0^* = \frac{2(-\psi_1^* + \Delta y^*)}{\lambda(\Delta y^*)^2}$$

where ψ_1^* is the stream function value in the flow field which is Δx^* or Δy^* away from the boundary whose stream function has a value ψ_0^*. (See fig. 13.)

APPENDIX C

DESCRIPTION OF COMPUTER PROGRAMS

Program ISGPSM

The computer program for the numerical solution of an idealized spiral grooved pumping seal model (ISGPSM) consists of the MAIN program and the following subroutines:

SVCALC enters the input data and calculates the stream function and vorticity distributions and many of the constant terms used throughout the program.

INDIST generates the stream function initial distribution.

ZETAF generates the vorticity initial distribution.

VORTWL calculates the values of the vorticity along the moving and stationary boundary walls.

UVFUNC computes the u^* and v^* velocity profiles in the x^* and y^* directions, respectively.

PRESS calculates the static pressure field based on the stream function and vorticity distributions and then the normalized pressure field with reference to a predetermined point.

BCDUMP punches data in sequentially numbered absolute binary cards with a maximum of 22 words per card.

Program ISGPSM is structured to make use of the overlay feature of the computing machine loading system (IBLDR) which saves in auxiliary storage those sections of the program currently not being executed. The use of this feature thus provides for a maximum of 65 mesh points in both the x^*- and y^*-directions. Figure 14 is a chart of the overlay structure of the entire ISGPSM program.

The input to program ISGPSM is a comparatively few number of variables which are entered on one data card. The following is a list of the variables and the definition of each:

LAMBDA	aspect ratio
RE	Reynolds number
OMEGA	relaxation factor [0, 2.0]

PCT maximum allowable relative change between two successive iterations of the stream function or static pressure field calculations at any mesh point

NOPTY number of mesh points in y*-direction (maximum number = 65)

NOPTX number of mesh points in x*-direction (maximum number = 65)

NOLINY number of lines of output data in y*-direction (maximum number = NOPTY)

NOLINX number of columns of output data in x*-direction (maximum number = 15)

NOLINY and NOLINX should be chosen such that (NOPTY - 1)/(NOLINY - 1) = K_y and (NOPTX - 1)/(NOLINX - 1) = K_x where K_x and K_y are integers. The program will then print lines 1, $1 + K_y$, $1 + 2K_y$, . . . , NOPTY and columns 1, $1 + K_x$, $1 + 2K_x$, . . ., NOPTX.

IREF mesh point number in the y*-direction for predetermined reference point in the normalized pressure field calculations

JREF mesh point number in the x*-direction for the predetermined reference point in the normalized pressure field calculations.

NOW a control word for the w*-velocity calculations; if NOW equals any nonzero integer, the converged values of the stream function at all mesh points will be punched into cards for use in w*-velocity profile program, but if NOW equals zero, no card punching will occur.

NOPLOT a control word for the normalized static pressure field plots. If NOPLOT equals any nonzero integer, the normalized static pressure field is punched using program BCDUMP for use with Canright and Swigert (ref. 6) three dimensional plotting program, but if NOPLOT equals zero, no card punching will occur.

PROBNO a control word for the type of problem to be solved. If PROBNO $\neq 0$, the program will solve Laplace's equation ($\zeta^* = 0$) or Poisson's equation ($\zeta^* = f(x, y)$). If PROBNO = 0 and Re = 0, a solution to the biharmonic equation is obtained.

FLUFLG a control word for additional fluid flow calculations. If FLUFLG = 0, the u^* and v^* velocity profiles and pressure fields will be calculated.

The format for this card is (4F8.0, 10I3).

1 8	9 16	17 24	25 32	35	38	41	44	47	50	53	56	59	62	72	73 80
X.XXX	XXX.	X.XX	.XXX	XX	XX	XX	XX	XX	XX	X	X	X	X		

The printed output from the ISGPSM program includes the aspect ratio, Reynolds number, relaxation factor, maximum allowable relative change, number of iterations to convergence of the stream function, and the number of increments in the x^* and y^* directions. Also printed are paragraphs of data NOLINY lines long and NOLINX columns wide of the stream function and vorticity, the u^* and v^* profiles, and the static and normalized pressure fields with the number of iterations to convergence of the static pressure field. Following the normalized pressure field are the leading edge region and whole surface net forces.

Program WFIELD

Program WFIELD is a FORTRAN IV computer code for calculating the z^*-direction velocity profile w^* and the dimensionless net volume flow rate Q_z^* along the groove axis. LAMBDA, RE, OMEGA, PCT, NOPTY, NOPTX, NOLINY, and NOLINX are the first input variables to WFIELD and are entered on one data card. The definition of these variables is the same as for the first eight variables in the ISGPSM program and the format for this card is (4F6.0, 4I3)

1 6	7 12	13 18	19 24	27	30	33	36	37 72	73 80
X.XXX	XXX.	X.XX	.XXX	XX	XX	XX	XX		

The second set of input cards is the values of the stream function (PSI) at all mesh points. These cards are punched in the format (9F8.2) by the ISGPSM program when the variable NOW is not equal to zero. The number of cards required is dependent on the grid size with a maximum of nine data words per card. The dimensionless groove axial pressure gradient $\partial P^*/\partial x^*$, which in the program is denoted by DPDZ, is the last input variable. For a specific geometry and stream function distribution, the w^*-velocity profile and net volume flow may be calculated for several $\partial P^*/\partial z^*$ values; however, each value must be entered on a separate data card in the format (F8.0).

1 8	9 72	73 80
X.XXX		

The output from the WFIELD program is the aspect ratio, Reynolds number, relaxation factor, maximum relative change, number of iterations to convergence, number of increments in the x* and y* directions, and pressure gradient. Associated with a particular pressure gradient, the w*-velocity at the specified mesh points and the net volume flow along the groove axis are also printed.

Program GRAPH

The three-dimensional plots of the normalized pressure field are generated using program GRAPH, subroutine PSURF, and the PLOT3D package of subroutines of Canright and Swigert (ref. 6). (It is assumed that the users of program GRAPH have the appropriate Calcomp plotter available although this is not necessary to use programs ISGPSM and WFIELD.) The data to be plotted is read by the computer via the FORTRAN IV program GRAPH. The PLOT3D subroutines then analyze the data to establish the array of coordinates of each point to be plotted, scale these values to the size of the plotting paper, and set up the figure axes. If the axes are to be rotated to present a nonstandard three dimensional projection, the data points and figure axes are transformed to the new coordinate system. The information on the data points, figure axes, and figure labels is then written on a magnetic tape for further processing by a California Computer Products (Calcomp) magnetic tape plotting system.

The input to program GRAPH is LAMBDA, NOPTY, NOPTX, and

NOYPLS number of planes parallel to x*-P* plane or perpendicular to the y*-axis

NOXPLS number of planes parallel to y*-P* plane or perpendicular to the x*-axis

XSCALF, YSCALF, PSCALF $\left.\right\}$ x*-, y*- and P*-direction scale factors used to adjust the coordinates of each point to match the size of the plotting paper and present the plot in the proper perspective (These scale factors are dependent on the amount of hardware on the Calcomp plotter.)

These variables are entered on one data card in the format (F6.0, 4I3, 3F6.0).

1 6	7 9	10 12	13 15	16 18	19 24	25 30	31 36	37 72	73 80
X.XXX	XX	XX	XX	XX	X.X	X.X	X.X		

In addition, the normalized pressure distribution at all mesh points is read from punched cards via the BCREAD input routine. These cards are generated in subroutine PRESS of the ISGPM and are punched by the BCDUMP routine in column binary format.

APPENDIX D

PROGRAM LISTING

```
$IBFTC MAIN    LIST,DECK
C
COMMENT--NUMERICAL SOLUTIONS OF CONVECTIVE INERTIA EFFECTS FOR AN
C         IDEALIZED, RECTANGULAR, PARALLEL GROOVED PUMP - SEAL MODEL.
C
      COMMON PSI(65,65),ZETA(65,65),NOPTX,NOPTY,NPTXM1,NPTYM1,DX,DY,
     1 TOODXS,TOODYS,LAMBDA,TWODX,TWODY,RE,XINDX,YINDX,PCT,IREF,JREF,
     2 U(65,65),V(65,65),NOW,NOPLOT,FLUFLG,PARTSI,LMODXS,LMDYSQ,PROBNO
      INTEGER FLUFLG
    1 CALL SVCALC
      IF (FLUFLG .EQ. 0) CALL PRESS
      GO TO 1
      END
```

```
$IBFTC CALCSV  LIST,DECK
      SUBROUTINE SVCALC
C
COMMENT -- STREAM AND VORTICITY DISTRIBUTIONS.
C
      COMMON PSI(65,65),ZETA(65,65),NOPTX,NOPTY,NPTXM1,NPTYM1,DX,DY,
     1 TOODXS,TOODYS,LAMBDA,TWODX,TWODY,RE,XINDX,YINDX,PCT,IREF,JREF,
     2 U(65,65),V(65,65),NOW,NOPLOT,FLUFLG,PARTSI,LMODXS,LMDYSQ,PROBNO
      DIMENSION PSIOUT(65,65)
      REAL LAMBDA,LMODXS,LMDYSQ,LMSDYS
      INTEGER XINDX,YINDX,PROBNO,FLUFLG
      LOGICAL JAIL
      WRITE (6,100)
  100 FORMAT (1H18X,38HIDEALIZED SPIRAL GROOVED PUMPING SEAL./1X)
   75 READ (5,3) LAMBDA,RE,OMEGA,PCT,NOPTY,NOPTX,NOLINY,NOLINX,IREF,JREF
     A ,NOW,NOPLOT,PROBNO,FLUFLG
    3 FORMAT(4F8.0,10I3)
      NPTXM1= NOPTX-1
      NPTYM1= NOPTY-1
  210 XINDX = NPTXM1/(NOLINX-1)
      YINDX = NPTYM1/(NOLINY-1)
      NPTXP1 = NOPTX+1
      NPTYP1 = NOPTY + 1
      DX = 1./FLOAT(NPTXM1)
      DY = 1./FLOAT(NPTYM1)
      DXSQ = DX * DX
      DYSQ = DY * DY
      TWODY = DY + DY
      TWODX = DX + DX
      LMODXS= LAMBDA/DXSQ
      LMDYSQ = LAMBDA*DYSQ
      LMSDYS = LAMBDA*LMDYSQ
      RE4DXY = RE/4./DX/DY
      PSIMLT = LMDYSQ*DXSQ/2./(LMSDYS+DXSQ)
      ZETMLT = LAMBDA*PSIMLT
```

```
      TOODYS = 2./DYSQ
      TOODXS = 2./DXSQ
      PARTSI =-2.*(LAMBDA/DXSQ + 1./LAMBDA/DYSQ)
      PARTZE = PARTSI/LAMBDA
      OMOPSI = OMEGA/PARTSI
      OMOPZE = OMEGA/PARTZE
      TERM = OMOPZE *RE4DXY
      ONEMOM= 1.-OMEGA
C
COMMENT--STREAM FUNCTION INITIAL DISTRIBUTION.
C
      CALL INDIST
C
COMMENT--VORTICITY INITIAL DISTRIBUTION.
C
      CALL ZETAF
   15 ITKONT = 0
C
COMMENT--ITERATIVE SOLUTIONS.
C
   17 ITKONT= ITKONT+1
      JAIL= .FALSE.
      IF (PROBNO .NE. 0) GO TO 21
      CALL VORTWL
      DO 20 J=2,NPTXM1
      DO 20 I=2,NPTYM1
      ZETA(I,J) = ZETA(I,J)*ONEMOM - OMOPZE*((ZETA(I,J+1)+ZETA(I,J-1))/
     1 DXSQ + (ZETA(I+1,J)+ZETA(I-1,J))/LMSDYS)
   20 IF (RE .GT. 0.)ZETA(I,J) = ZETA(I,J)-TERM*((PSI(I,J+1)-PSI(I,J-1))
     1 *(ZETA(I-1,J)-ZETA(I+1,J)) - (PSI(I-1,J)-PSI(I+1,J))*(ZETA(I,J+1)
     2 -ZETA(I,J-1)))
   21 DO 22 J=2,NPTXM1
      DO 22 I=2,NPTYM1
      PSIF= PSI(I,J)*ONEMOM - OMOPSI*(LMODXS*(PSI(I,J+1)+PSI(I,J-1))+
     1 (PSI(I+1,J)+PSI(I-1,J))/LMDYSQ + ZETA(I,J))
      IF (JAIL) GO TO 22
   23 IF (ABS((PSIF-PSI(I,J))/PSIF) .GT. PCT) JAIL = .TRUE.
   22 PSI(I,J)= PSIF
      IF (JAIL) GO TO 17
   45 WRITE (6,46) LAMBDA,RE,OMEGA,PCT,ITKONT,NPTXM1,NPTYM1
   46 FORMAT (9X,8HLAMBDA =F6.3,5X,4HRE =E12.4,5X, 19HRELAXATION FACTOR
     A=F5.2/9X,24HMAXIMUM RELATIVE ERROR =F9.6,5X,22HNUMBER OF ITERATION
     BS =I5/9X,37HNUMBER OF INCREMENTS IN X-DIRECTION =I4,5X,16HIN Y-DIR
     CECTION =I4/1H08X,24HSTREAM FUNCTION X 10E+4,/1X)
C
COMMENT--STREAM FUNCTION AND VORTICITY OUTPUT .
C
      DO 50 J=1,NOPTX
      DO 50 I=1,NOPTY
   50 PSIOUT(I,J)=PSI(I,J)*1.E+4
      IF (NOW .NE. 0) PUNCH 48,((PSIOUT(I,J),I=1,NOPTY),J=1,NOPTX)
   48 FORMAT(9F8.2)
      DO 51 I=1,NOPTY,YINDX
   51 WRITE (6,52) (PSIOUT(I,J),J=1,NOPTX,XINDX)
   52 FORMAT(9X,15F8.2)
      WRITE (6,64)
   64 FORMAT(1H18X,10HVORTICITY,/1X)
      DO 67 I=1,NOPTY,YINDX
   67 WRITE (6,68) (ZETA(I,J),J=1,NOPTX,XINDX)
```

```
   68 FORMAT(9X,15F8.3)
      IF (FLUFLG .EQ. 0) CALL UVFUNC
      RETURN
      END
```

```
$IBFTC DISTI    LIST,DECK
      SUBROUTINE INDIST
      COMMON PSI(65,65),ZETA(65,65),NOPTX,NOPTY,NPTXM1,NPTYM1,DX,DY,
     1 TOODXS,TOODYS,LAMBDA,TWODX,TWODY,RE,XINDX,YINDX,PCT,IREF,JREF,
     2 U(65,65),V(65,65),NOW,NOPLOT,FLUFLG,PARTSI,LMODXS,LMDYSQ,PROBNO
      NPTXP1 = NOPTX+1
      NPTYP1 = NOPTY+1
      MIDPTX = NPTXP1/2
      DO 8 J = 1,MIDPTX
      PSITRM = -.2*FLOAT(J-1)*DX
      MAX = NPTXP1 -J
      DO 8 I=1,NOPTY
      PSI(I,J)=PSITRM
    8 PSI(I,MAX)=PSITRM
      MIDPTY=NPTYP1/2
      JMIN=0
      JMAX=NPTXP1
      DO 10 I=1,MIDPTY
      PSITRM = -.2*FLOAT(I-1)*DY
      MAX = NPTYP1 - I
      JMIN = JMIN+1
      JMAX = JMAX -1
      IF (JMIN .LE. JMAX) GO TO 9
      JMIN = NPTXP1/2
      JMAX = (NPTXP1 + 1)/2
    9 DO 10 J=JMIN,JMAX
      PSI(I,J)=PSITRM
   10 PSI(MAX,J)=PSITRM
      RETURN
      END
```

```
$IBFTC ZETAXY   LIST,DECK
      SUBROUTINE ZETAF
      COMMON PSI(65,65),ZETA(65,65),NOPTX,NOPTY,NPTXM1,NPTYM1,DX,DY,
     1 TOODXS,TOODYS,LAMBDA,TWODX,TWODY,RE,XINDX,YINDX,PCT,IREF,JREF,
     2 U(65,65),V(65,65),NOW,NOPLOT,FLUFLG,PARTSI,LMODXS,LMDYSQ,PROBNO
      REAL LMODXS,LMDYSQ
   14 DO 16 J=2,NPTXM1
      DO 16 I=2,NPTYM1
   16 ZETA(I,J)=-PARTSI*PSI(I,J)-(LMODXS*(PSI(I,J+1)+PSI(I,J-1)) +
     1(PSI(I+1,J)+PSI(I-1,J))/LMDYSQ)
      DO 12 J=1,NOPTX,NPTXM1
      DO 12 I=1,NOPTY,NPTYM1
   12 ZETA(I,J) =0.0
      RETURN
      END
```

```
$IBFTC VRTWAL  LIST,DECK
      SUBROUTINE VORTWL
C
COMMENT--VORTICITY BOUNDARY VALUES.
C
      COMMON PSI(65,65),ZETA(65,65),NOPTX,NOPTY,NPTXM1,NPTYM1,DX,DY,
     1 TOODXS,TOODYS,LAMBDA,TWODX,TWODY,RE,XINDX,YINDX,PCT,IREF,JREF,
     2 U(65,65),V(65,65),NOW,NOPLOT,FLUFLG,PARTSI,LMODXS,LMDYSQ,PROBNO
      REAL LAMBDA
      DO 10 J=2,NPTXM1
C
COMMENT--MOVING WALL.
C
      ZETA(1,J) =       (PSI(1,J)-PSI(2,J)-DY)*TOODYS/LAMBDA
C
COMMENT--STATIONARY WALLS.
C
   10 ZETA(NOPTY,J) = (PSI(NOPTY,J)-PSI(NPTYM1,J))*TOODYS/LAMBDA
      DO 20 I=2,NPTYM1
      ZETA(I,1) = (PSI(I,1)-PSI(I,2))*TOODXS*LAMBDA
   20 ZETA(I,NOPTX) = (PSI(I,NOPTX)-PSI(I,NPTXM1))*TOODXS*LAMBDA
      RETURN
      END
```

```
$IBFTC FUNCUV  LIST,DECK
      SUBROUTINE UVFUNC
C
COMMENT -- U* AND V* VELOCITY DISTRIBUTIONS.
C
      COMMON PSI(65,65),ZETA(65,65),NOPTX,NOPTY,NPTXM1,NPTYM1,DX,DY,
     1 TOODXS,TOODYS,LAMBDA,TWODX,TWODY,RE,XINDX,YINDX,PCT,IREF,JREF,
     2 U(65,65),V(65,65),NOW,NOPLOT,FLUFLG,PARTSI,LMODXS,LMDYSQ,PROBNO
      INTEGER XINDX,YINDX
  101 DO 102 I=1,NOPTY
      U(I,1)=0.0
      V(I,1)=0.0
      U(I,NOPTX)=0.0
  102 V(I,NOPTX)=0.0
      DO 104 J=1,NOPTX
      U(1,J) =  1.0
      V(1,J) =  0.0
      U(NOPTY,J) = 0.0
  104 V(NOPTY,J) = 0.0
      DO 106 J=2,NPTXM1
      DO 106 I=2,NPTYM1
      U(I,J) = (PSI(I-1,J) - PSI(I+1,J))/TWODY
  106 V(I,J) = -(PSI(I,J+1)-PSI(I,J-1))/TWODX
      WRITE (6,1000)
 1000 FORMAT(1H18X,20HU* VELOCITY PROFILE,/1X)
      DO 1002 I=1,NOPTY,YINDX
 1002 WRITE (6,1004) (U(I,J),J=1,NOPTX,XINDX)
 1004 FORMAT(9X,15F8.4)
      WRITE (6,1007)
 1007 FORMAT(1H18X,20HV* VELOCITY PROFILE,/1X)
      DO 1010 I=1,NOPTY,YINDX
 1010 WRITE (6,1004) (V(I,J),J=1,NOPTX,XINDX)
      RETURN
      END
```

```
$IBFTC PCALC   LIST,DECK
      SUBROUTINE PRESS
C
COMMENT--PRESSURE FIELD CALCULATION.
C
      COMMON PSI(65,65),ZETA(65,65),NOPTX,NOPTY,NPTXM1,NPTYM1,DX,DY,
     1 TOODXS,TOODYS,LAMBDA,TWODX,TWODY,RE,XINDX,YINDX,PCT,IREF,JREF,
     2 U(65,65),V(65,65),NOW,NOPLOT,FLUFLG,PARTSI,LMODXS,LMDYSQ,PROBNO
      DIMENSION XTERM(65,65),YTERM(65,65),P(65,65),PNORM(65,65)
      EQUIVALENCE (P,PSI),(PNORM,ZETA)
      INTEGER XINDX,YINDX
      INTEGER PROBNO
      LOGICAL JAIL
      REAL LAMBDA
      REAL LEREG
      FOURDY = TWODY + TWODY
      FOURDX = TWODX + TWODX
      NPTXM2 = NPTXM1-1
      NPTYM2 = NPTYM1 - 1
      NPTXM3 = NPTXM2-1
      NPTYM3 = NPTYM2-1
C
COMMENT -- INTERIOR POINTS.
C
      CON2 =  LAMBDA/2.
      CON3 =  TWODX*FOURDY
      CON4 = (LAMBDA/DX)**2/2.
      CON6 = TWODY*DY
      CON7 = LAMBDA**2/CON3
      IF (RE .GT. 1.) GO TO 7
      CON2 = CON2*RE
      CON3 = CON3*RE
      CON4 = CON4*RE
      CON6 = CON6*RE
      CON7 = CON7*RE
      CON1 = -1./LAMBDA/FOURDY
      CON5 = LAMBDA/FOURDX
      GO TO 8
    7 CON1 = -1./RE/LAMBDA/FOURDY
      CON5 = LAMBDA/RE/FOURDX
    8 DO 10 I=2,NPTYM1
      DO   9 J=3,NPTXM1
      DIF1 = PSI(I-1,J+1)-PSI(I-1,J-1)+PSI(I+1,J-1)-PSI(I+1,J+1)
    9 XTERM(I,J) = DX*(CON1*(ZETA(I-1,J)-ZETA(I+1,J)+ZETA(I-1,J-1)-ZETA
     A(I+1,J-1))+CON2*(V(I,J)*ZETA(I,J)+V(I,J-1)*ZETA(I,J-1)) - (U(I,J)*
     BDIF1+U(I,J-1)*(PSI(I-1,J)-PSI(I-1,J-2)+PSI(I+1,J-2)-PSI(I+1,J)))/
     CCON3 + CON4*(V(I,J)*(PSI(I,J+1)-2.*PSI(I,J)+PSI(I,J-1))+V(I,J-1)*
     D(PSI(I,J)-2.*PSI(I,J-1)+PSI(I,J-2))))
   10 XTERM(I,2)=XTERM(I,3)
      DO 20 J=2,NPTXM1
      DO 19 I=2,NPTYM2
      DIF1 = PSI(I-1,J+1)-PSI(I-1,J-1)+PSI(I+1,J-1)-PSI(I+1,J+1)
   19 YTERM(I,J) = DY*(CON5*(ZETA(I,J+1)-ZETA(I,J-1)+ZETA(I+1,J+1)-ZETA
     A(I+1,J-1))-CON2*(U(I,J)*ZETA(I,J)+U(I+1,J)*ZETA(I+1,J))- (U(I,J)*
     B(PSI(I-1,J)-2.*PSI(I,J)+PSI(I+1,J))+U(I+1,J)*(PSI(I,J)-2.*PSI(I+1,
     CJ)+PSI(I+2,J)))/CON6 + CON7*(V(I,J)*DIF1+V(I+1,J)*(PSI(I,J+1)-PSI
     D(I,J-1)+PSI(I+2,J-1)-PSI(I+2,J+1))))
   20 YTERM(NPTYM1,J)=YTERM(NPTYM2,J)
C
```

```
COMMENT -- VERTICAL BOUNDARY WALL POINTS.
C
      CON3 = CON3/2.
      CON5 = CON5*2.
      CON7 = 2.*CON7
      DO 30 I=2,NPTYM1
      XTERM(I,1) = DX*(CON1*(ZETA(I-1,1)-ZETA(I+1,1)+ZETA(I-1,2)-ZETA
     A (I+1,2)) +CON2*(ZETA(I,1)*V(I,1)+ZETA(I,2)*V(I,2)) - (U(I,1)*
     B (PSI(I-1,2)-PSI(I-1,1)+PSI(I+1,1)-PSI(I+1,2))+U(I,2)*(PSI(I-1,3)
     C -PSI(I-1,2)+PSI(I+1,2)-PSI(I+1,3)))/CON3 + CON4*(V(I,1)*(PSI(I,3)
     D -2.*PSI(I,2)+PSI(I,1))+V(I,2)*(PSI(I,4)-2.*PSI(I,3)+PSI(I,2))))
   30 XTERM(I,NOPTX) = DX*(CON1*(ZETA(I-1,NOPTX)-ZETA(I+1,NOPTX)+ZETA
     A (I-1,NPTXM1)-ZETA(I+1,NPTXM1)) +CON2*(ZETA(I,NOPTX)*V(I,NOPTX)+
     B ZETA(I,NPTXM1)*V(I,NPTXM1)) -(U(I,NOPTX)*(PSI(I-1,NOPTX)-PSI(I-1,
     C NPTXM1)+PSI(I+1,NPTXM1)-PSI(I+1,NOPTX))+U(I,NPTXM1)*(PSI(I-1,
     D NPTXM1)-PSI(I-1,NPTXM2)+PSI(I+1,NPTXM2)-PSI(I+1,NPTXM1)))/CON3
     E + CON4*(V(I,NOPTX)*(PSI(I,NOPTX)-2.*PSI(I,NPTXM1)+PSI(I,NPTXM2))
     F + V(I,NPTXM1)*(PSI(I,NPTXM1)-2.*PSI(I,NPTXM2)+PSI(I,NPTXM3))))
      DO 40 I=2,NPTYM2
      YTERM(I,1) = DY*(CON5*(ZETA(I,2)-ZETA(I,1)+ZETA(I+1,2)-ZETA(I+1,1)
     A ) - CON2*(ZETA(I,1)*U(I,1)+ZETA(I+1,1)*U(I+1,1)) - (U(I,1)*(PSI
     B (I-1,1)-2.*PSI(I,1)+PSI(I+1,1))+U(I+1,1)*(PSI(I,1)-2.*PSI(I+1,1)+
     C PSI(I+2,1)))/CON6 + CON7*(V(I,1)*(PSI(I-1,2)-PSI(I-1,1)+PSI(I+1,
     D 1)-PSI(I+1,2))+V(I+1,1)*(PSI(I,2)-PSI(I,1)+PSI(I+2,1)-PSI(I+2,2))
     E ))
   40 YTERM(I,NOPTX) = DY*(CON5*(ZETA(I,NOPTX)-ZETA(I,NPTXM1)+ZETA(I+1,
     A NOPTX)-ZETA(I+1,NPTXM1))- CON2*(ZETA(I,NOPTX)*U(I,NOPTX)+ZETA(I+1
     B ,NOPTX)*U(I+1,NOPTX)) - (U(I,NOPTX)*(PSI(I-1,NOPTX)-2.*PSI(I,
     C NOPTX)+PSI(I+1,NOPTX))+U(I+1,NOPTX)*(PSI(I,NOPTX)-2.*PSI(I+1,
     D NOPTX)+PSI(I+2,NOPTX)))/CON6 + CON7*(V(I,NOPTX)*(PSI(I-1,NOPTX)-
     E PSI(I-1,NPTXM1)+PSI(I+1,NPTXM1)-PSI(I+1,NOPTX))+V(I+1,NOPTX)*(PSI
     F (I,NOPTX)-PSI(I,NPTXM1)+PSI(I+2,NPTXM1)-PSI(I+2,NOPTX))))
      YTERM(NPTYM1,1) = YTERM(NPTYM2,1)
      YTERM(NPTYM1,NOPTX) = YTERM(NPTYM2,NOPTX)
C
COMMENT -- HORIZONTAL BOUNDARY WALL POINTS.
C
      IF(PROBNO .NE. 0)  GO TO 49
      ZETA(1,1) =-2./DY/LAMBDA
      ZETA(1,NOPTX) = ZETA(1,1)
   49 CON1 = 2.*CON1
      CON5 = CON5/2.
      DO 50 J=3,NPTXM1
      XTERM(1,J) = DX*(CON1*(ZETA(1,J)-ZETA(2,J)+ZETA(1,J-1)-ZETA(2,J-1)
     A )+CON2*(ZETA(1,J)*V(1,J)+ZETA(1,J-1)*V(1,J-1)) - (U(1,J)*(PSI(1,
     B J+1)-PSI(1,J-1)+PSI(2,J-1)-PSI(2,J+1))+U(1,J-1)*(PSI(1,J)-PSI(1,
     C J-2)+PSI(2,J-2)-PSI(2,J)))/CON3 + CON4*(V(1,J)*(PSI(1,J+1)-2.*PSI
     D (1,J)+PSI(1,J-1))+V(1,J-1)*(PSI(1,J)-2.*PSI(1,J-1)+PSI(1,J-2))))
   50 XTERM(NOPTY,J) = DX*(CON1*(ZETA(NPTYM1,J)-ZETA(NOPTY,J)+ZETA(
     A NPTYM1,J-1)-ZETA(NOPTY,J-1))+CON2*(ZETA(NOPTY,J)*V(NOPTY,J)+ZETA
     B (NOPTY,J-1)*V(NOPTY,J-1)) - (U(NOPTY,J)*(PSI(NPTYM1,J+1)-PSI(
     C NPTYM1,J-1)+PSI(NOPTY,J-1)-PSI(NOPTY,J+1))+U(NOPTY,J-1)*(PSI(
     D NPTYM1,J)-PSI(NPTYM1,J-2)+PSI(NOPTY,J-2)-PSI(NOPTY,J)))/CON3+CON4
     E *(V(NOPTY,J)*(PSI(NOPTY,J+1)-2.*PSI(NOPTY,J)+PSI(NOPTY,J-1))+V(
     F NOPTY,J-1)*(PSI(NOPTY,J)-2.*PSI(NOPTY,J-1)+PSI(NOPTY,J-2))))
      XTERM(1,2)=XTERM(1,3)
      XTERM(NOPTY,2)=XTERM(NOPTY,3)
      DO 60 J=2,NPTXM1
      YTERM(1,J) = DY*(CON5*(ZETA(1,J+1)-ZETA(1,J-1)+ZETA(2,J+1)-ZETA(2,
     A J-1))-CON2*
```

```
      *          (ZETA(1,J)*U(1,J)+ZETA(2,J)*U(2,J))-(U(1,J)*(PSI(1,J)-2.*
      B PSI(2,J)+PSI(3,J))+U(2,J)*(PSI(2,J)-2.*PSI(3,J)+PSI(4,J)))/CON6 +
      C CON7*(V(1,J)*(PSI(1,J+1)-PSI(1,J-1)+PSI(2,J-1)-PSI(2,J+1))+V(2,J)
      D *(PSI(2,J+1)-PSI(2,J-1)+PSI(3,J-1)-PSI(3,J+1))))
   60 YTERM(NOPTY,J) = DY*(CON5*(ZETA(NOPTY,J+1)-ZETA(NOPTY,J-1)+ZETA
      A (NPTYM1,J+1)-ZETA(NPTYM1,J-1))-CON2*(ZETA(NOPTY,J)*U(NOPTY,J)+
      B ZETA(NPTYM1,J)*U(NPTYM1,J))-(U(NOPTY,J)*(PSI(NPTYM2,J)-2.*PSI(
      C NPTYM1,J)+PSI(NOPTY,J))+U(NPTYM1,J)*(PSI(NPTYM3,J)-2.*PSI(NPTYM2,
      D J)+PSI(NPTYM1,J)))/CON6+CON7*(V(NOPTY,J)*(PSI(NPTYM1,J+1)-PSI(
      E NPTYM1,J-1)+PSI(NOPTY,J-1)-PSI(NOPTY,J+1))+V(NPTYM1,J)*(PSI(
      F NPTYM2,J+1)-PSI(NPTYM2,J-1)+PSI(NPTYM1,J-1)-PSI(NPTYM1,J+1))))
C
COMMENT -- CORNER POINTS.
C
       CON3 = CON3/2.
       CON5 = CON5*2.
       CON7 = CON7*2.
       DIF1 = PSI(1,2)-PSI(1,1)+PSI(2,1)-PSI(2,2)
       XTERM(1,1) = CON2*(ZETA(1,1)*V(1,1)+ZETA(1,2)*V(1,2))-(U(1,1)*DIF1
      A                                   +U(1,2)*(PSI(1,3)-PSI(1,2)+PSI
      B (2,2)-PSI(2,3)))/CON3+CON4*(V(1,1)*(PSI(1,3)-2.*PSI(1,2)+PSI(1,1)
      C )+V(1,2)*(PSI(1,4)-2.*PSI(1,3)+PSI(1,2)))
       YTERM(1,1) = CON5*(ZETA(1,2)-ZETA(1,1)+ZETA(2,2)-ZETA(2,1))-(U(1,1
      A )*(PSI(1,1)-2.*PSI(2,1)+PSI(3,1))+U(2,1)*(PSI(2,1)-2.*PSI(3,1)+
      B PSI(4,1)))/CON6+CON7*(V(1,1)*DIF1
      C  +V(2,1)*(PSI(2,2)-PSI(2,1)+PSI(3,1)-PSI(3,2)))
       DIF1 = PSI(1,NOPTX)-PSI(1,NPTXM1)+PSI(2,NPTXM1)-PSI(2,NOPTX)
       XTERM(1,NOPTX) = CON2*(ZETA(1,NPTXM1)*V(1,NPTXM1)+ZETA(1,NOPTX)*V
      A (1,NOPTX))-(U(1,NOPTX)*DIF1+U(1,NPTXM1)*(PSI(1,NPTXM1)-PSI(1,
      B NPTXM2)+PSI(2,NPTXM2)-PSI(2,NPTXM1)))/CON3+CON4*(V(1,NOPTX)*(PSI
      C (1,NOPTX)-2.*PSI(1,NPTXM1)+PSI(1,NPTXM2))+V(1,NPTXM1)*(PSI(1,
      D NPTXM1)-2.*PSI(1,NPTXM2)+PSI(1,NPTXM3)))
       YTERM(1,NOPTX)= CON5*(ZETA(1,NOPTX)-ZETA(1,NPTXM1)+ZETA(2,NOPTX)-
      A ZETA(2,NPTXM1))-(U(1,NOPTX)*(PSI(1,NOPTX)-2.*PSI(2,NOPTX)+PSI(3,
      B NOPTX))+U(2,NOPTX)*(PSI(2,NOPTX)-2.*PSI(3,NOPTX)+PSI(4,NOPTX)))/
      C CON6+CON7*(V(1,NOPTX)*DIF1+V(2,NOPTX)*(PSI(2,NOPTX)-PSI(2,NPTXM1)
      D +PSI(3,NPTXM1)-PSI(3,NOPTX)))
       IF(PROBNO .NE. 0) GO TO 70
       ZETA(1,1)= 0.0
       ZETA(1,NOPTX) = 0.0
   70 XTERM(1,1)= DX*(CON1*(ZETA(1,1)-ZETA(2,1)+ZETA(1,2)-ZETA(2,2)) +
      A XTERM(1,1))
       YTERM(1,1)= DY*(YTERM(1,1)-CON2*(ZETA(1,1)*U(1,1)+ZETA(2,1)*U(2,1
      A )))
       XTERM(1,NOPTX) = DX*(CON1*(ZETA(1,NOPTX)-ZETA(2,NOPTX)+ZETA(1,
      A NPTXM1)- ZETA(2,NPTXM1))+XTERM(1,NOPTX))
       YTERM(1,NOPTX)= DY*(YTERM(1,NOPTX)-CON2*(ZETA(1,NOPTX)*U(1,NOPTX)
      A +ZETA(2,NOPTX)*U(2,NOPTX)))
       DIF1= PSI(NPTYM1,2)-PSI(NPTYM1,1)+PSI(NOPTY,1)-PSI(NOPTY,2)
       XTERM(NOPTY,1) = DX*(CON1*(ZETA(NPTYM1,1)-ZETA(NOPTY,1)+ZETA(
      A NPTYM1,2) -ZETA(NOPTY,2))+CON2*(ZETA(NOPTY,1)*V(NOPTY,1)+ZETA(
      B NOPTY,2)*V(NOPTY,2))-(U(NOPTY,1)*DIF1+U(NOPTY,2)*(PSI(NPTYM1,3)-
      C PSI(NPTYM1,2)+PSI(NOPTY,2)-PSI(NOPTY,3)))/CON3+ CON4*(V(NOPTY,1)
      D *(PSI(NOPTY,3)-2.*PSI(NOPTY,2)+PSI(NOPTY,1))+V(NOPTY,2)*(PSI(
      E NOPTY,4)-2.*PSI(NOPTY,3)+PSI(NOPTY,2))))
       YTERM(NOPTY,1) = DY*(CON5*(ZETA(NOPTY,2)-ZETA(NOPTY,1)+ZETA(NPTYM1
      A ,2)-ZETA(NPTYM1,1))-CON2*(ZETA(NOPTY,1)*J(NOPTY,1)+ZETA(NPTYM1,1)
      B *U(NPTYM1,1))-(U(NOPTY,1)*(PSI(NPTYM2,1)-2.*PSI(NPTYM1,1)+PSI(
      C NOPTY,1))+U(NPTYM1,1)*(PSI(NPTYM3,1)-2.*PSI(NPTYM2,1)+PSI(NPTYM1,
      D 1)))/CON6+CON7*(V(NOPTY,1)*DIF1+V(NPTYM1,1)*(PSI(NPTYM2,2)-PSI(
```

```
      E NPTYM2,1)+PSI(NPTYM1,1)-PSI(NPTYM1,2))))
       DIF1= PSI(NPTYM1,NOPTX)-PSI(NPTYM1,NPTXM1)+PSI(NOPTY,NPTXM1)-PSI
      A (NOPTY,NOPTX)
       XTERM(NOPTY,NOPTX)= DX*(CON1*(ZETA(NPTYM1,NOPTX)-ZETA(NOPTY,NOPTX)
      A +ZETA(NPTYM1,NPTXM1)-ZETA(NOPTY,NPTXM1))+CON2*(ZETA(NOPTY,NOPTX)*
      B V(NOPTY,NOPTX)+ZETA(NOPTY,NPTXM1)*V(NOPTY,NPTXM1))-(U(NOPTY,NOPTX
      C )*DIF1+U(NOPTY,NPTXM1)*(PSI(NPTYM1,NPTXM1)-PSI(NPTYM1,NPTXM2)+PSI
      D (NOPTY,NPTXM2)-PSI(NOPTY,NPTXM1)))/CON3+CON4*(V(NOPTY,NOPTX)*(PSI
      E (NOPTY,NOPTX)-2.*PSI(NOPTY,NPTXM1)+PSI(NOPTY,NPTXM2))+V(NOPTY,
      F NPTXM1)*(PSI(NOPTY,NPTXM1)-2.*PSI(NOPTY,NPTXM2)+PSI(NOPTY,NPTXM3)
      G )))
       YTERM(NOPTY,NOPTX)= DY*(CON5*(ZETA(NOPTY,NOPTX)-ZETA(NOPTY,NPTXM1
      A )+ZETA(NPTYM1,NOPTX)-ZETA(NPTYM1,NPTXM1))-CON2*(ZETA(NOPTY,NOPTX)
      B *U(NOPTY,NOPTX)-ZETA(NPTYM1,NOPTX)*U(NPTYM1,NOPTX))-(U(NOPTY,
      C NOPTX)*(PSI(NPTYM2,NOPTX)-2.*PSI(NPTYM1,NOPTX)+PSI(NOPTY,NOPTX))+
      D U(NPTYM1,NOPTX)*(PSI(NPTYM3,NOPTX)-2.*PSI(NPTYM2,NOPTX)+PSI(
      E NPTYM1,NOPTX)))/CON6+CON7*(V(NOPTY,NOPTX)*DIF1+V(NPTYM1,NOPTX)*
      F (PSI(NPTYM2,NOPTX)-PSI(NPTYM2,NPTXM1)+PSI(NPTYM1,NPTXM1)-PSI(
      G NPTYM1,NOPTX))))
C
COMMENT--INITIALIZE PRESSURE FIELD.
C
      DO 200 J=1,NOPTX
      DO 200 I=1,NOPTY
  200 P(I,J)= 1.0
      ITKONT= 0
  202 ITKONT= ITKONT+1
      JAIL= .FALSE.
      DO 210 I=1,NPTYM2
      DO 210 J=1,NOPTX
  204 IF (J .GT. 2) GO TO 205
      PX= P(I,J+1)-XTERM(I,J)
      GO TO 207
  205 PX= P(I,J-1)+XTERM(I,J)
  207 PNEW= 0.5*(PX+P(I+1,J)+YTERM(I,J))
      IF (JAIL) GO TO 209
      IF (ABS((PNEW-P(I,J))/PNEW) .GT.    PCT) JAIL = .TRUE.
  209 P(I,J) = PNEW
  210 CONTINUE
      DO 220 I=NPTYM1,NOPTY
      DO 220 J=1,NOPTX
  214 IF (J .GT. 2) GO TO 215
      PX = P(I,J+1)-XTERM(I,J)
      GO TO 217
  215 PX = P(I,J-1)+XTERM(I,J)
  217 PNEW = 0.5*(PX+P(I-1,J)-YTERM(I,J))
      IF (JAIL) GO TO 219
      IF (ABS((PNEW-P(I,J))/PNEW) .GT.    PCT) JAIL =.TRUE.
  219 P(I,J) = PNEW
  220 CONTINUE
      IF ( .NOT. JAIL) GO TO 229
      IF(ITKONT .LT. 500) GO TO 202
      WRITE (6,225)
  225 FORMAT(1H14X,50H*****NO CONVERGED SOLUTION IN 500 ITERATIONS.*****)
     A)
  229 WRITE (6,230)
  230 FORMAT(1H18X,18HPRESSURE FIELD, P*)
      IF (RE .LE. 1.) WRITE (6,231)
  231 FORMAT(1H+26X,1H')
      WRITE (6,232) ITKONT
```

```
232 FORMAT(1H08X,19HNO. OF ITERATIONS =I4/1X)
    DO 235 I=1,NOPTY,YINDX
235 WRITE (6,106) (P(I,J),J=1,NOPTX,XINDX)
106 FORMAT(9X,15F8.4)
    IF (JAIL) GO TO 202
    DO 240 J=1,NOPTX
    DO 240 I=1,NOPTY
240 PNORM(I,J)= P(I,J)-P(IREF,JREF)
    WRITE (6,241)
241 FORMAT(1H18X,26HNORMALIZED PRESSURE FIELD,/1X)
    DO 245 I=1,NOPTY,YINDX
245 WRITE (6,106) (PNORM(I,J),J=1,NOPTX,XINDX)
    DXO3 = DX/3.
    NPTXP1 = NOPTX+1
    DO 250 I=1,NOPTX
    II = NPTXP1 - I
    IF (PNORM(1,II) .LT. 0.) GO TO 252
250 CONTINUE
252 II = II+1
    LEREG = 0.
    IF (MOD(II,2) .NE. 0) GO TO 255
    LEREG = (PNORM(1,II)+PNORM(1,II+1))*DX/2.
    II = II+1
255 IF (II.EQ. NOPTX) GO TO 263
    DO 260 J=II,NPTXM2,2
260 LEREG = (PNORM(1,J)+4.*PNORM(1,J+1)+PNORM(1,J+2))*DXO3 + LEREG
263 WHSURF = 0.
    DO 265 J=1,NPTXM2,2
265 WHSURF = WHSURF + (PNORM(1,J)+4.*PNORM(1,J+1)+PNORM(1,J+2))*DXO3
    WRITE (6,505) LEREG,WHSURF
505 FORMAT(1HK8X,35HNET FORCE FOR LEADING EDGE REGION =1PE13.5/23X,15H
   AWHOLE SURFACE =E13.5)
    IF (NOPLOT .EQ. 0) RETURN
    DO 333J=1,NOPTX
333 CALL BCDUMP(PNORM(1,J),PNORM(NOPTY,J),1)
    RETURN
    END
```

```
$IBMAP BCDUMP
       TTL      BCDUMP ROUTINE FOR IBSYS
       ENTRY    BCDUMP
BCDUMP SAVE     1,2,4
       CLA      1,4              IS THERE A
       PDX      0,2                THIRD
       TXI      *+2,2,2              ARGUMENT
       NZT*     5,4              YES, IS IT = 0
       SXA      CNUM,0                YES
       LDQ      3,4
       TLQ      *+2
       XCA
       STQ      ..BRDB           ..BRDB HAS THE FIRST ADDRESS
       LXA      ..BRDB,1         PUT FIRST LOC. IN IX1
       SUB      ..BRDB
       PAX      0,2              THE NO. OF WORDS OUTPUTED IN INDEX 2
       TXI      *+1,2,1          TRUE WORD COUNT
       SXA      IX1,1
```

```
         SXA     IX2,2
IX1      AXT     **,1
IX2      AXT     **,2
         TSX     ..FTCK,4
TEST     TNX     LASTC,2,22
         SXA     IX2,2
         AXT     22,2
TEST4    TXI     *+1,2,320
         SXD     ..BRDB,2
         TIX     *+1,2,320
         SXA     CLA,1
         SXA     ..BRDB,1
LOOP     TXI     *+1,1,22
         SXA     IX1,1
         AXT     27,4
CLEAR    STZ     ..BRDB+28,4     CLEAR THE BUFFER.
         TIX     *-1,4,1
         AXT     0,4
CLA      CLA     **,4            FILL THE BUFFER WITH
         STO     ..BRDB+2,4
         TXI     *+1,4,-1
         TIX     *-3,2,1
CNUM     AXT     **,1            CONSECUTIVELY
         CLA     HUNBIT          NUMBER
         ARS     1               THE
         TXL     *+2,1,99        BCDUMP
         TXI     *-2,1,-100      CARDS
         STA     GP+1            FROM
         CLA     BITT            ZERO
         ARS     1               TO
         TXL     *+2,1,9         999
         TXI     *-2,1,-10
         STO     WORD3
         CLA     BITU
         ARS     1
         TXL     *+2,1,0
         TXI     *-2,1,-1
         ORS     WORD3
         LXA     CNUM,1
         TXI     *+1,1,1
         TXL     *+2,1,999
         AXT     0,1
         SXA     CNUM,1
         DLD     GP
         DST     ..BRDB+24
         DLD     GP+2
         DST     ..BRDB+26
         AXT     22,1            COMPUTE
         CAL     ..BRDB          THE
         ACL     ..BRDB+24       CHECK SUM
         TIX     *-1,1,1
         SLW     ..BRDB+1
         AXC     OUT-3,1
         SXA     SYSLOC,1
         CALL    ..BCWD
         TSX     ..FIOC,4
         TRA     IX1
RETURN   TRA     *+1
         AXT     IX1,1
```

```
         SXA        RETURN-1,1
         TSX        ..FTCK,4
         RETURN     BCDUMP
LASTC    CLA        RETURN
         STO        RETURN-1
         TRA        TEST4
BP       OCT        420041004040
         OCT        104020400000
WORD3    PZE
         PZE
HUNBIT   OCT        2000
BITU     OCT        20000000
BITT     OCT        200000000000
OUT      PZE        .PCH.
         *LDIR
         END
```

```
$IBFTC WFIELD  LIST,DECK
C
COMMENT -- W* VELOCITY PROFILES
C
      DIMENSION PSI(65,65),W(65,65),DPSIDY(65,65),DPSIDX(65,65),
     A WPCT(65,65)
      REAL LAMBDA
      INTEGER XINDX,YINDX
      LOGICAL INDKT
      READ (5,3) LAMBDA,RE,OMEGA,PCT,NOPTY,NOPTX,NOLINY,NOLINX
    3 FORMAT(4F6.0,4I3)
      NPTXM1 = NOPTX-1
      NPTYM1 = NOPTY-1
      XINDX = NPTXM1/(NOLINX-1)
      YINDX = NPTYM1/(NOLINY-1)
      BSQDSQ= 1./LAMBDA**2
      DX = 1./FLOAT(NPTXM1)
      DY = 1./FLOAT(NPTYM1)
      DXSQ= DX*DX
      DYSQ= DY*DY
      TWODX= DX+DX
      TWODY= DY+DY
      WPF = 2.*(1./DXSQ + BSQDSQ/DYSQ)/RE
      ONMOM = 1. - OMEGA
      OMOWPF= OMEGA/WPF
C
COMMENT -- SET UP STREAM FUNCTION DISTRIBUTION
C
      READ (5,5) ((PSI(I,J) ,I=1,NOPTY),J=1,NOPTX)
    5 FORMAT (9F8.0)
      DO 8 J=1,NOPTX
      DO 8 I=1,NOPTY
    8 PSI(I,J)= PSI(I,J)*1.E-4
      DO 12 J=2,NPTXM1
      DO 12 I=2,NPTYM1
      DPSIDY(I,J)= (PSI(I-1,J)-PSI(I+1,J))/TWODY
   12 DPSIDX(I,J)= (PSI(I,J+1)-PSI(I,J-1))/TWODX
C
```

```
COMMENT -- INITIALIZE W* FIELD
C
      DO 14 J=2,NPTXM1
      W(NOPTY,J) = 0.0
      DO 14 I=1,NPTYM1
   14 W(I,J)= -1.0
      W(1,1)= -1.0
      W(1,NOPTX) = -1.0
      DO 16 I=2,NOPTY
      W(I,1)= 0.0
   16 W(I,NOPTX) = 0.0
   17 WRITE (6,28)
   28 FORMAT(1H1)
      READ (5,18) DPDZ
   18 FORMAT( F8.0)
      ITKONT = 0
C
COMMENT -- ITERATE W* FIELD FOR GIVEN DP/DZ VALUE.
C
   20 INDKT = .FALSE.
      ITKONT = ITKONT + 1
      DO 22 I=2,NPTYM1
      DO 22 J=2,NPTXM1
      WF= DPSIDY(I,J)*(W(I,J+1)-W(I,J-1))/TWODX - DPSIDX(I,J)*(W(I-1,J)
     A-W(I+1,J))/TWODY+DPDZ-((W(I,J+1)+W(I,J-1))/DXSQ+BSQDSQ*(W(I+1,J)+
     BW(I-1,J))/DYSQ)/RE
      WNEW = ONMOM*W(I,J) - OMOWPF*WF
      WPCT(I,J)= (W(I,J)-WNEW)/WNEW
      IF (INDKT) GO TO 22
   25 IF(ABS(WPCT(I,J)) .GT. PCT) INDKT = .TRUE.
   22 W(I,J)= WNEW
      IF (INDKT) GO TO 20
C
COMMENT -- CALCULATE Q,NET FROM CONVERGED W* FIELD.
C
      QNET = 0.
      DO 60 I=2,NPTYM1,2
      DO 60 J=2,NPTXM1,2
   60 QNET = QNET+16.*W(I,J)+4.*(W(I-1,J)+W(I+1,J)+W(I,J-1)+W(I,J+1))
     A +W(I-1,J-1)+W(I-1,J+1)+W(I+1,J-1)+W(I+1,J+1)
      QNET = QNET*DX*DY/9.
C
COMMENT -- OUTPUT
C
      WRITE (6,26) LAMBDA,RE,OMEGA,PCT,ITKONT,NPTXM1,NPTYM1,DPDZ,QNET
   26 FORMAT(    9X,8HLAMBDA =F6.3,5X,4HRE =F7.1,5X,19HRELAXATION FACTOR
     A=F5.2/9X,24HMAXIMUM RELATIVE ERROR =F6.3,5X,22HNUMBER OF ITERATION
     BS =I5/9X,37HNUMBER OF INCREMENTS IN X-DIRECTION =I4,5X,16HIN Y-DIR
     CECTION =I4/9X,9HDP*/DZ* =F7.3,6X,6HQ*,Z = 1PE13.5/1H08X,20HW* VELO
     DCITY PROFILE,/1X)
      DO 30 I=1,NOPTY,YINDX
   30 WRITE (6,33) (W(I,J),J=1,NOPTX,XINDX)
   33 FORMAT(1X,15F8.4)
      GO TO 17
      END
```

```
$IBFTC GRAPH    LIST,DECK
      COMMON LAMBDA,NOPTY,NOPTX,NOYPLS,NOXPLS,DY,DX,PNORM
      COMMON /SKALF /XSCALF,YSCALF,ZSCALF
      DIMENSION Z(13000),PNORM(65,65)
      EXTERNAL PSURF
      REAL LAMBDA
      READ (5,4) LAMBDA,NOPTY,NOPTX,NOYPLS,NOXPLS,XSCALF,YSCALF,PSCALF
    4 FORMAT(F6.0,4I3,3F6.0)
      ZSCALF = PSCALF
      DY= 1./FLOAT(NOPTY-1)
      DX= 1./FLOAT(NOPTX-1)
      NOPTS = 3*(NOPTX*NOYPLS+NOPTY*NOXPLS)
      DO 10 J=1,NOPTX
   10 CALL BCREAD(PNORM(1,J),PNORM(NOPTY,J))
      CALL PLOT3D (0.,1.,0.,LAMBDA,Z,NOPTX,NOYPLS,NOXPLS,NOPTY,PSURF,
     A .TRUE.)
      CALL ROTATE (0.,0.,45.,.FALSE.)
      CALL ROTATE (0.,35.,0.,.TRUE.)
      STOP
      END
```

```
$IBFTC SURFAC  LIST,DECK
      FUNCTION PSURF(I,J)
      COMMON LAMBDA,NOPTY,NOPTX,NOYPLS,NOXPLS,DY,DX,PNORM(65,65)
      II= NOPTY+1 -I
      PSURF = PNORM(II,J)
      RETURN
      END
```

```
$IBMAP .BCREA
        ENTRY    BCREAD
BCREAD  SAVE     1,4
        CLA      3,4               GET FIRST ARG.
        LDQ      4,4               GET SECOND ARG.
        TLQ      *+2               COMPARE
        XCA                        IF 2ND LESS  EXCHANGE
        STQ      TEMP              STORE SMALLEST ARG
        ADD      SYSONE            ADD 1
        STA      STO               STORE FOR MOVE
        SUB      TEMP              COMPUTE COUNT
        STA      IX1               STORE FOR MOVE
        AXC      UNC-3,4           LOCATE UN05 LIKE FIV CALL
        SXA      SYSLOC,4          AND SAVE IN SYSLOC
        CALL     ..BCRD            SET UP READ
READ    TSX      ..FIOC,4          READ RECORD
        TSX      ..FTCK,4          CHECK READ
IX1     AXT      **,1              PICK UP COUNT LEFT
        TXL      LASTC,1,22        IS ONLY 1 REC LEFT
IX4     AXT      0,4               REC CNT
        CLA      ..BRDB+2,4        MOVE WORDS
STO     STO      **,1              TO STORE
        TIX      *+1,1,1           DECR. COUNT
        TXI      *+1,4,-1          DECR. REC COUNT
```

```
CKIR4   TXH      STO-1,4,-22         CR. REC COUNT
        SXA      IX1,1              NO SAVE REMAINING COUNT
        TRA      READ               GO READ NEXT RECORD
LASTC   TXL      DONE,1,0           ANY MORE WORDS
        AXT      DONE,4             YES STORE TO EXIT NEXT
        SXA      LASTC-1,4          TIME
        SCD      CKIR4,1            SET REC CNT = NO WORDS LEFT
        TRA      IX4                GO PROCESS RECORD
DONE    AXT      READ,4             RESTORE EXIT
        SXA      LASTC-1,4
        AXT      -22,4              RESTORE REC CNT
        SXD      CKIR4,4
        RETURN   BCREAD
UNC     PZE      .UN05.             ADD OF UNIT 5
TEMP    PZE
        *LDIR
        END
```

```
$IBMAP  .BCRWD
        ENTRY    ..BROB
        ENTRY    ..BCRD
        ENTRY    ..BCWD
SIZE    SET      23                 RECORD SIZE
..BCRD  CLA      PON                READ ENTRY FOR BCREAD
        TRA      *+2                GET CORRECT ARG FOR .FIOS
..BCWD  CAL      PTH                WRITE ENTRY FOR BCDUMP
        SXA      LK.DR,4            SAVE IR4
        CALL     ..FIOS(SEL)        SET UP READ OR WRITE
        ORG      *-1
SEL     IORT     ..BROB,,SIZE         I/O COMMAND
        LXA      LK.DR,4            RESTORE 4
        TRA      1,4
PON     PON      0,,0
PTH     PTH      0,,0
..BROB  BSS      SIZE               I/O BUFFER
LK.DR   LDIR
        END
```

APPENDIX E

SAMPLE PROBLEM

A sample problem, which was solved on the Lewis IBM 7044-7094 direct-couple system, is included to show the user how to set up the input data cards and to show what output can be expected. For this problem LAMBDA = 1 (square groove), RE = 100, OMEGA = 1.3, PCT = 0.001, NOPTY and NOPTX = 29, and NOLINY and NOLINX = 15. The reference mesh point for the normalized pressure field calculations is (29, 15), and the punched output for the w*-velocity calculations and normalized pressure field plot is required. The control words PROBNO and FLUFLG are zero in this problem. Results for this case are discussed in reference 1. The input data card for the ISGPSM program should be punched as

1 8	9 16	17 24	25 32	35	38	41	44	47	50	53	56	59	62	63 72	73 80
1.000	100.0	1.30	0.001	29	29	15	15	29	15	1	1	0	0		

The printed output for this problem is shown in table I. The number of iterations printed in table I(a) is the number of iterations required for convergence of the stream function to within the designated relative change. It should be noted that the values of the stream function have been multiplied by 10^4 before printout to facilitate writing the output format of the stream function and to present more significant figures in the individual values. In table I(b) the values of the vorticity at the four corners are those calculated from the equations for the vertical stationary walls. If the values at the four corners based on the equations for horizontal walls are desired, they are $\zeta^* = 0$ for the lower stationary wall and $\zeta^* = -2/\lambda \Delta y^*$ for the upper moving wall. The static pressure field is printed in table I(e) along with the number of iterations to convergence to within the designated relative change. The relative change for convergence of the pressure field is equal to that of the stream function. The normalizing factor for this particular problem is 1.0025 and is the entry found in the eighth column of the bottom row. This number is used to calculate the normalized pressure field distribution (table I(f)). Also shown in table I(f) is the leading edge force and net force acting on the moving surface.

Using the punched output of the stream function from the ISGPSM program, the w*-field and Q_z^* can now be calculated using the WFIELD program. The additional data

TABLE I. - SAMPLE PROBLEM OUTPUT

(a) Stream function

[illegible] SPIRAL GROOVED PUMPING SEAL.

[illegible] = 1.000 RE = 0.1000E+03 RELAXATION FACTOR = 1.30
MAXIMUM ITERATION ERROR = 0.001000 NUMBER OF ITERATIONS = 185
NUMBER OF INCREMENTS IN X-DIRECTION = 28 IN Y-DIRECTION = 28

STREAM FUNCTION X 10E+4,

-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.
-0.	[illegible]	-270.96	-363.21	-427.54	-474.68	-510.06	-535.78	-551.78	-555.34	-545.20	-513.45	-449.77	-295.67	-0.
-0.	[illegible]	-254.95	-404.23	-531.79	-638.19	-725.14	-792.48	-837.23	-853.52	-831.27	-754.04	-539.90	-237.91	-0.
-0.	-72.[illegible]1	-221.25	-395.86	-542.41	-582.41	-801.85	-895.18	-958.21	-975.81	-935.52	-810.79	-571.71	-225.13	-0.
-0.	-51.11	-135.73	-350.25	-522.75	-674.38	-805.85	-908.59	-971.78	-980.05	-913.39	-750.03	-431.25	-154.63	-0.
-0.	[illegible]	-175.36	-326.51	-482.85	-630.09	-757.47	-853.80	-905.44	-895.79	-805.35	-525.30	-353.92	-113.70	-0.
-0.	[illegible]	-152.34	-283.64	-425.44	-557.06	-658.97	-748.79	-782.42	-754.80	-654.33	-473.93	-253.12	-74.83	-0.
-0.	-33.14	-125.49	-239.21	-355.26	-454.65	-554.87	-613.95	-629.40	-590.08	-491.20	-342.49	-175.34	-47.70	-0.
-0.	-27.61	[illegible]	-185.97	-279.12	-363.98	-471.24	-470.59	-472.39	-430.14	-344.77	-229.60	-112.74	-23.44	-0.
-0.	-21.15	-71.59	-136.39	-204.10	-265.50	-312.11	-335.91	-330.35	-232.63	-225.60	-144.37	-57.95	-15.18	-0.
-0.	-13.47	-45.77	-93.37	-136.13	-177.14	-207.09	-220.32	-212.82	-193.94	-133.04	-84.95	-37.90	-3.35	-0.
-0.	-7.16	-25.20	-51.98	-79.37	-103.83	-121.24	-128.04	-121.95	-103.17	-75.11	-44.25	-13.43	-3.51	-0.
-0.	-2.68	-11.15	-23.32	-36.54	-48.41	-56.79	-59.85	-55.45	-45.85	-32.97	-18.23	-5.57	-0.73	-0.
-0.	[illegible]	-2.44	-5.78	-9.51	-12.90	-15.33	-15.23	-15.23	-12.38	-8.31	-4.12	-1.02	0.20	-0.
-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.

(b) Vorticity

VORTICITY,

0.	-35.240	-22.726	-16.292	-12.539	-10.065	-8.330	-7.145	-6.489	-6.439	-7.153	-8.919	-12.510	-21.559	0.
[illegible]	[illegible]	-5.433	-6.311	-6.417	-6.233	-5.960	-5.690	-5.482	-5.391	-5.500	-5.998	-7.453	-9.453	23.434
[illegible]	1.419	-0.367	-1.365	-2.041	-2.546	-2.961	-3.338	-3.710	-4.106	-4.565	-5.193	-5.344	-1.779	15.588
[illegible]	[illegible]	0.472	-0.299	-0.875	-1.378	-1.858	-2.376	-2.917	-3.493	-4.084	-4.532	-3.731	1.974	11.096
2.722	[illegible]	0.554	-0.118	-0.635	-1.105	-1.584	-2.095	-2.638	-3.175	-3.582	-3.430	-1.532	3.153	7.399
[illegible]	1.328	0.513	-0.074	-0.535	-0.961	-1.403	-1.866	-2.322	-2.680	-2.727	-2.002	0.039	3.081	4.314
[illegible]	1.145	0.493	-0.033	-0.418	-0.774	-1.137	-1.497	-1.795	-1.915	-1.544	-0.707	0.929	2.512	3.026
[illegible]	0.959	0.409	0.022	-0.275	-0.542	-0.800	-1.025	-1.155	-1.092	-0.674	0.135	1.152	1.344	1.830
1.208	0.759	0.365	0.085	-0.125	-0.304	-0.459	-0.565	-0.571	-0.411	-0.037	0.510	1.034	1.257	1.059
[illegible]	[illegible]	0.324	0.143	0.012	-0.092	-0.171	-0.202	-0.153	0.005	0.258	0.574	0.795	0.807	0.573
[illegible]	0.416	0.282	0.189	0.125	0.078	0.047	0.048	0.099	0.207	0.355	0.497	0.559	0.483	0.273
[illegible]	0.272	0.240	0.221	0.215	0.211	0.209	0.216	0.242	0.291	0.348	0.385	0.370	0.275	0.091
[illegible]	0.160	0.198	0.238	0.283	0.320	0.343	0.350	0.347	0.339	0.323	0.292	0.235	0.141	-0.005
[illegible]	0.078	0.150	0.239	0.336	0.421	0.477	0.493	0.465	0.403	0.315	0.220	0.132	0.055	-0.027
0.	[illegible]	0.079	0.222	0.384	0.532	0.540	0.582	0.539	0.513	0.330	0.144	0.013	-0.025	0.

(c) u*-Velocity profile

U* VELOCITY PROFILE,

1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.	[illegible]	0.0827	0.2049	0.3124	0.4016	0.4739	0.5295	0.5559	0.5316	[illegible]	[illegible]	0.3775	0.1228	0.
0.	-0.0471	-0.0481	-0.0035	0.0592	0.1248	0.1854	0.2357	0.2701	0.2618	0.2503	0.1934	0.0550	-0.0575	0.
0.	-0.0231	-0.0403	-0.0334	-0.0102	0.0203	0.0508	0.0753	0.0873	0.0819	0.0301	-0.0121	-0.0874	-0.0895	0.
0.	-0.0125	-0.0309	-0.0411	-0.0422	-0.0360	-0.0333	-0.0325	-0.0408	-0.0619	-0.0970	-0.1375	-0.1435	-0.0794	0.
0.	-0.0101	-0.0306	-0.0513	-0.0687	-0.0855	-0.0981	-0.1152	-0.1359	-0.1634	-0.1889	-0.1963	-0.1562	-0.0629	0.
0.	-0.0107	-0.0343	-0.0622	-0.0904	-0.1177	-0.1446	-0.1717	-0.1981	-0.2197	-0.2255	-0.2027	-0.1357	-0.0462	0.
0.	-0.0115	-0.0377	-0.0699	-0.1037	-0.1371	-0.1592	-0.1983	-0.2211	-0.2314	-0.2195	-0.1757	-0.1051	-0.0321	0.
0.	-0.0125	-0.0388	-0.0720	-0.1070	-0.1411	-0.1720	-0.1971	-0.2115	-0.2102	-0.1852	-0.1331	-0.0755	-0.0213	0.
0.	-0.0114	-0.0368	-0.0682	-0.1010	-0.1319	-0.1582	-0.1764	-0.1825	-0.1727	-0.1443	-0.1004	-0.0515	-0.0137	0.
0.	[illegible]	-0.0320	-0.0595	-0.0879	-0.1138	-0.1342	-0.1458	-0.1453	-0.1321	-0.1052	-0.0695	-0.0340	-0.0087	0.
0.	-0.0076	-0.0251	-0.0472	-0.0701	-0.0905	-0.1055	-0.1123	-0.1091	-0.0953	-0.0728	-0.0450	-0.0215	-0.0052	0.
0.	-0.0046	-0.0167	-0.0325	-0.0492	-0.0640	-0.0745	-0.0785	-0.0747	-0.0633	-0.0464	-0.0277	-0.0120	-0.0025	0.
0.	-0.0018	-0.0077	-0.0164	-0.0257	-0.0345	-0.0406	-0.0427	-0.0402	-0.0332	-0.0231	-0.0125	-0.0043	-0.0004	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

TABLE I. - Concluded. SAMPLE PROBLEM OUTPUT

(d) v*-Velocity profile

V* VELOCITY PROFILE,

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.2130	0.1567	0.1972	0.0767	0.0572	0.0426	0.0294	0.0148	-0.0332	-0.0272	-0.0534	-0.1350	-0.3380	0.
0.	0.2005	0.2201	0.1842	0.1634	0.1352	0.1023	0.0733	0.0441	-0.0020	-0.0550	-0.1511	-0.3100	-0.4931	0.
0.	0.1534	0.2257	0.2271	0.2034	0.1823	0.1575	0.1110	0.0580	-0.0124	-0.1113	-0.2495	-0.4015	-0.4595	0.
0.	0.1400	0.2149	0.2315	0.2223	0.1995	0.1657	0.1194	0.0530	-0.0357	0.1555	-0.3043	-0.4372	-0.3815	0.
0.	0.1510	0.1766	0.2194	0.2145	0.1941	0.1547	0.1052	0.0325	-0.0550	-0.1574	-0.3135	-0.3703	-0.2331	0.
0.	0.1135	0.1724	0.1941	0.1920	0.1725	0.1365	0.0920	0.0053	[illegible]	-0.1944	-0.2624	-0.2091	-0.1055	0.
0.	0.0941	0.1410	0.1627	0.1606	0.1417	0.1057	0.0543	-0.0151	-0.0956	-0.1753	-0.2240	-0.2155	-0.1293	0.
0.	0.0734	0.1132	0.1284	0.1257	0.1083	0.0764	0.0303	-0.0277	-0.0934	-0.1440	-0.1543	-0.1455	-0.0800	0.
0.	0.0500	0.0820	0.0945	0.0919	0.0771	0.0506	0.0137	-0.0303	-0.0740	-0.1054	-0.1150	-0.0932	-0.0479	0.
0.	.0345	0.0554	0.0634	0.0610	0.0507	0.0311	0.0045	-0.0257	-0.0535	-0.0714	-0.0725	-0.0551	-0.0252	0.
0.	0.0181	0.0223	0.0381	0.0370	0.0300	0.0175	0.0007	-0.0177	-0.0337	-0.0425	-0.0410	-0.0292	-0.0124	0.
0.	0.0082	0.0140	0.0182	0.0180	0.0145	0.0083	-0.0001	-0.0093	-0.0169	-0.0207	-0.0190	-0.0125	-0.0042	0.
0.	.0017	0.0040	0.0051	0.0051	0.0042	0.0024	-0.0000	-0.0027	-0.0050	-0.0060	-0.0053	-0.0031	-0.0005	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

(e) Pressure field

PRESSURE FIELD, P*

NO. OF ITERATIONS 30

[illegible]	[illegible]	1.3417	[illegible]	1.1200	1.0504	1.0142	[illegible]	0.9500	0.9617	0.9932	1.0701	1.2215	1.3303	2.2525
[illegible]	0.9345	0.9431	0.9550	0.9606	0.9527	0.9505	0.9377	0.9300	0.9349	0.9515	1.0220	1.1349	1.3359	1.4913
0.9173	0.9102	0.9375	0.9514	0.9551	0.9501	0.9396	0.9275	0.9135	0.9222	0.9433	0.9929	1.0740	1.1577	1.1535
0.9932	0.9413	0.9457	0.9510	0.9500	0.9441	0.9341	0.9229	0.9147	0.9155	0.9317	0.9583	1.0217	1.0581	1.0443
0.9578	0.9550	0.9557	0.9553	0.9522	0.9455	0.9361	0.9251	0.9192	0.9200	0.9334	0.9615	0.9978	1.0215	1.0035
0.9595	0.9575	0.9658	0.9635	0.9592	0.9527	0.9447	0.9370	0.9325	0.9348	0.9471	0.9589	0.9925	1.0025	0.9888
0.9796	0.9775	0.9754	0.9727	0.9687	0.9635	0.9576	0.9527	0.9510	0.9549	0.9557	0.9814	0.9945	0.9953	0.9858
0.9855	0.9859	0.9838	0.9816	0.9785	0.9749	0.9714	0.9591	0.9595	0.9740	0.9824	0.9910	0.9975	0.9955	0.9877
0.9992	0.9914	0.9900	0.9890	0.9870	0.9849	0.9832	0.9827	0.9843	0.9883	0.9934	0.9985	0.9998	0.9965	0.9912
0.9996	0.9970	0.9955	0.9944	0.9933	0.9923	0.9918	0.9923	0.9940	0.9970	1.0001	1.0019	1.0013	0.9983	0.9947
[illegible]	0.9997	0.9985	0.9977	0.9972	0.9969	0.9971	0.9971	0.9995	1.0015	1.0031	1.0035	1.0023	1.0001	0.9977
1.0012	1.0015	0.9998	0.9993	0.9991	0.9993	0.9998	1.0008	1.0022	1.0035	1.0044	1.0043	1.0032	1.0016	1.0001
1.0011	1.0008	1.0000	0.9997	0.9997	1.0001	1.0009	1.0021	1.0034	1.0046	1.0052	1.0049	1.0040	1.0029	1.0022
1.0017	1.0032	0.9995	0.9991	0.9992	0.9999	1.0010	1.0025	1.0041	1.0054	1.0060	1.0057	1.0048	1.0039	1.0035
1.0002	0.9995	0.9985	0.9979	0.9980	0.9988	1.0004	1.0025	1.0047	1.0065	1.0073	1.0070	1.0058	1.0044	1.0039

(f) Normalized pressure field

NORMALIZED PRESSURE FIELD,

[illegible]	0.4545	0.3392	0.2061	0.1175	0.0569	0.0118	-0.0218	-0.0415	-0.0409	-0.0093	0.0575	0.2100	0.5778	1.2501
-0.1743	-0.1175	-0.0593	-0.0365	-0.0320	-0.0338	-0.0520	-0.0548	-0.0724	-0.0575	-0.0409	0.0135	0.1324	0.3344	0.4839
-0.0502	-0.0432	-0.0450	-0.0511	-0.0474	-0.0524	-0.0629	-0.0749	-0.0830	-0.0803	-0.0585	-0.0095	0.0715	0.1552	0.1512
-0.0532	-0.0512	-0.0558	-0.0522	-0.0525	-0.0583	-0.0684	-0.0795	-0.0877	-0.0870	-0.0704	-0.0342	0.0132	0.0555	0.0413
-0.0457	-0.0456	-0.0467	-0.0471	-0.0503	-0.0570	-0.0664	-0.0753	-0.0833	-0.0825	-0.0590	-0.0410	-0.0047	0.0191	0.0010
-0.0339	-0.0349	-0.0367	-0.0390	-0.0433	-0.0497	-0.0577	-0.0554	-0.0700	-0.0575	-0.0534	-0.0335	-0.0095	0.0001	-0.0137
-0.0224	-0.0250	-0.0271	-0.0298	-0.0337	-0.0390	-0.0449	-0.0498	-0.0515	-0.0475	-0.0357	-0.0211	-0.0079	-0.0051	-0.0157
-0.0140	-0.0155	-0.0126	-0.0209	-0.0239	-0.0275	-0.0311	-0.0334	-0.0329	-0.0284	-0.0201	-0.0105	-0.0049	-0.0070	-0.0147
-0.0073	-0.0100	-0.0118	-0.0135	-0.0154	-0.0175	-0.0193	-0.0197	-0.0182	-0.0142	-0.0087	-0.0039	-0.0025	-0.0058	-0.0113
-0.0028	-0.0054	-0.0070	-0.0081	-0.0092	-0.0102	-0.0107	-0.0102	-0.0084	-0.0055	-0.0024	-0.0005	-0.0012	-0.0041	-0.0078
-0.0005	-0.0028	-0.0040	-0.0048	-0.0053	-0.0056	-0.0054	-0.0045	-0.0020	-0.0010	0.0005	0.0010	-0.0001	-0.0024	-0.0045
0.0011	-0.0017	-0.0027	-0.0031	-0.0033	-0.0032	-0.0025	-0.0015	-0.0002	0.0011	0.0020	0.0018	0.0007	-0.0009	-0.0023
-0.0005	-0.0017	-0.0025	-0.0028	-0.0023	-0.0024	-0.0016	-0.0004	0.0010	0.0021	0.0027	0.0024	0.0015	0.0004	-0.0003
-0.0017	-0.0023	-0.0030	-0.0034	-0.0033	-0.0026	-0.0015	0.0000	0.0015	0.0029	0.0035	0.0032	0.0024	0.0014	0.0011
-0.0025	-0.0028	-0.0039	-0.0045	-0.0045	-0.0035	-0.0021	0.	0.0022	0.0040	0.0049	0.0045	0.0033	0.0019	0.0015

NET FORCE FOR LEADING EDGE REGION = 1.09637E-01
WHOLE SURFACE = 1.94249E-01

required are that LAMBDA = 1, RE = 100, PCT = 1.30, NOPTY = 29, NOPTX = 29, NOLINY = 15, NOLINX = 15, and $\partial P^*/\partial z^* = -0.115$. The first WFIELD program input data card is punched as

1 6	7 12	13 18	19 24	25 27	28 30	31 33	34 36	37 ... 72	73 80
1.000	100.0	1.30	.001	29	29	15	15		

This card is followed by the deck of cards containing the stream function values of ISGPSM and the card with the $\partial P^*/\partial z^*$ value.

1 8	9 ... 72	73 80
-0.115		

If more $\partial P^*/\partial z^*$ values are to follow for a given problem, they must be punched one to a card according to this last format.

Table II contains the one page of output per $\partial P^*/\partial z^*$ value from the WFIELD program. The number of iterations printed is the number of iterations of the w*-velocity profile from initial value to converged value within the prescribed maximum relative error. The net volume flow Q_z^* associated with the $\partial P^*/\partial z^*$ value of -0.115 in this problem is 0.0003128 and is printed on the same line as the $\partial P^*/\partial z^*$ value.

The computer execution time on the Lewis Computer for this sample problem was approximately a half a minute for each program.

TABLE II. - SAMPLE PROGRAM OUTPUT FOR WFIELD

```
[illegible] 1.[illegible]   [illegible] 100.[illegible]   RELAXATION FACTOR = 1.30
[illegible] = 0.001   NUMBER OF ITERATIONS = 315
[illegible] IN X-DIRECTION = 29   IN Y-DIRECTION = 29
[illegible] = [illegible]

W* VELOCITY PROFILE
```

-1.[illegible]	-1.[illegible]	[illegible]1.[illegible]	-1.[illegible]	-1.[illegible]	-1.[illegible]	-1.[illegible]	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000
0.	-0.[illegible]	-0.07[illegible]	[illegible].1325	-0.1[illegible]77	[illegible].2414	-0.2403	-0.3392	-0.3497	-0.4452	-0.5102	-0.5913	-0.6981	-0.7343	0.
0.	0.1512	0.1[illegible]	[illegible]	[illegible].1624	0.1191	0.0615	-0.0019	-0.0730	-0.1576	-0.2552	-0.4051	-0.5571	-0.5047	0.
0.	[illegible]	0.2421	[illegible].2701	0.24[illegible]	[illegible].1971	0.1372	0.0720	-0.0007	-0.0903	-0.2080	-0.3542	-0.4519	-0.3451	0.
0.	[illegible].15[illegible]	0.2[illegible]	[illegible].2775	[illegible]	[illegible].1272	0.1172	0.0456	-0.0291	-0.1155	-0.2218	-0.3317	-0.3590	-0.2345	0.
0.	[illegible].1[illegible]	0.25[illegible]	.2775	0.24[illegible]	[illegible].1696	0.0856	0.0089	-0.0584	-0.1433	-0.2307	-0.2915	-0.2750	-0.1541	0.
0.	[illegible].1[illegible]	0.2[illegible]	.2[illegible]	[illegible]	0.1[illegible]	0.0787	-0.0055	-0.0[illegible]23	-0.1512	-0.2055	-0.2277	-0.1873	-0.0923	0.
0.	.15[illegible]	0.2512	.2[illegible]3	[illegible].24[illegible]7	0.1790	0.0934	0.0114	-0.0502	-0.1153	-0.1490	-0.1430	-0.1059	-0.0424	0.
0.	.1[illegible]	0.25[illegible]7	.2[illegible]2	[illegible]	[illegible].030	0.1286	0.0555	0.0054	-0.0704	-0.0714	-0.0541	-0.0332	-0.0003	0.
0.	.1	0.2[illegible]	.2[illegible]	[illegible].751	[illegible].21[illegible]	0.1724	0.1145	0.0553	0.0307	0.0155	0.0122	0.0325	0.0353	0.
0.	.1[illegible]	0.[illegible]	[illegible]	.27[illegible]	[illegible].2[illegible]97	0.2100	0.1590	0.1335	0.1057	0.0748	0.0905	0.0557	0.0529	0.
0.	.1[illegible]7	0.2171	[illegible]	.2[illegible]	[illegible].2[illegible]4	0.2233	0.1985	0.1754	0.1591	0.1455	0.1355	0.1175	0.0735	0.
0.	.1[illegible]	.1[illegible]	[illegible]3	0.2140	0.2100	0.1963	0.1452	0.1743	0.1535	0.1538	0.1412	0.1135	0.0731	0.
0.	[illegible]	0.1[illegible]7	[illegible].1[illegible]4	[illegible].1516	[illegible].1303	0.1267	0.1216	0.1155	0.1117	0.1050	0.0977	0.0832	0.0555	0.
0.	.	0.	.	.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

The three-dimensional plot of the normalized pressure distribution is obtained from program GRAPH using the column binary punched output of the pressure distribution from the ISGPSM program. The additional information required is that LAMBDA = 1; NOPTY = 29, NOPTX = 29, NOYPLS = 15, NOXPLS = 15, and XCALF, YSCALF, PSCALF each equal 6.0. The first program input data card is punched as

1 6	7 9	10 12	13 15	16 18	19 24	25 30	31 36	37 72	73 80
1.000	29	29	15	15	6.00	6.00	6.00		

After this is the deck of cards containing the pressure distribution values.

The output for this particular problem is the graph found in figure 15. The maximum value of $P^* = 1.2501$ is found at the corner where the moving wall meets the leading edge and the minimum value of $P^* = -0.3304$ is near the corner formed by the moving wall and the trailing edge. Note also the shallow pressure drop or relative minimum in the vicinity of the vortex center.

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1. Zuk, John; and Renkel, Harold E.: Numerical Solutions for the Flow and Pressure Fields in an Idealized Spiral Grooved Pumping Seal. Proceedings of the Fourth International Conference on Fluid Sealing (also ASLE Special Publication SP-2), 1969, Paper FICFS-30.

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6. Canright, R. Bruce, Jr.; and Swigert, Paul: Plot 3D- A Package of FORTRAN Subprograms to Draw Three-Dimensional Surfaces. NASA TM X-1598, 1968.

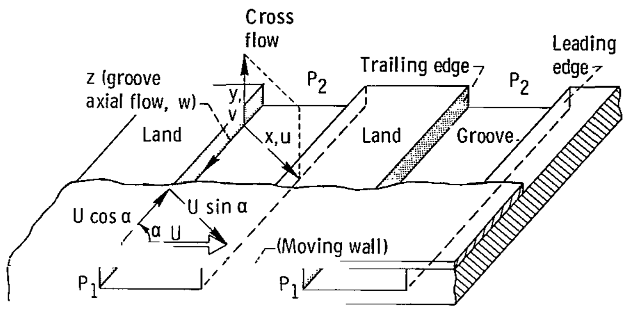

Figure 1. - Spiral groove pumping seal model for limiting case of land clearance.

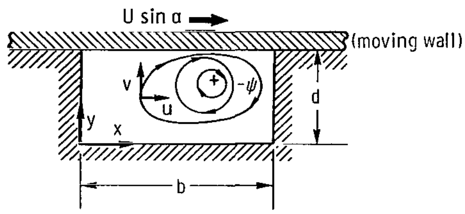

Figure 2. - Streamlines in groove cross flow plane. (+ designates vortex center.)

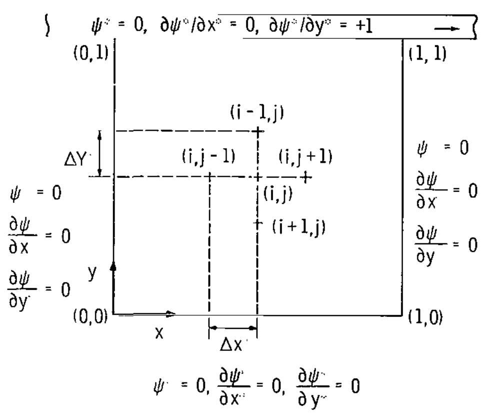

Figure 3. - Mesh point representation of x^*-y^* plane flow field.

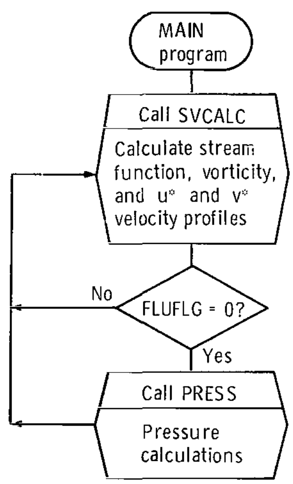

Figure 4. - MAIN program.

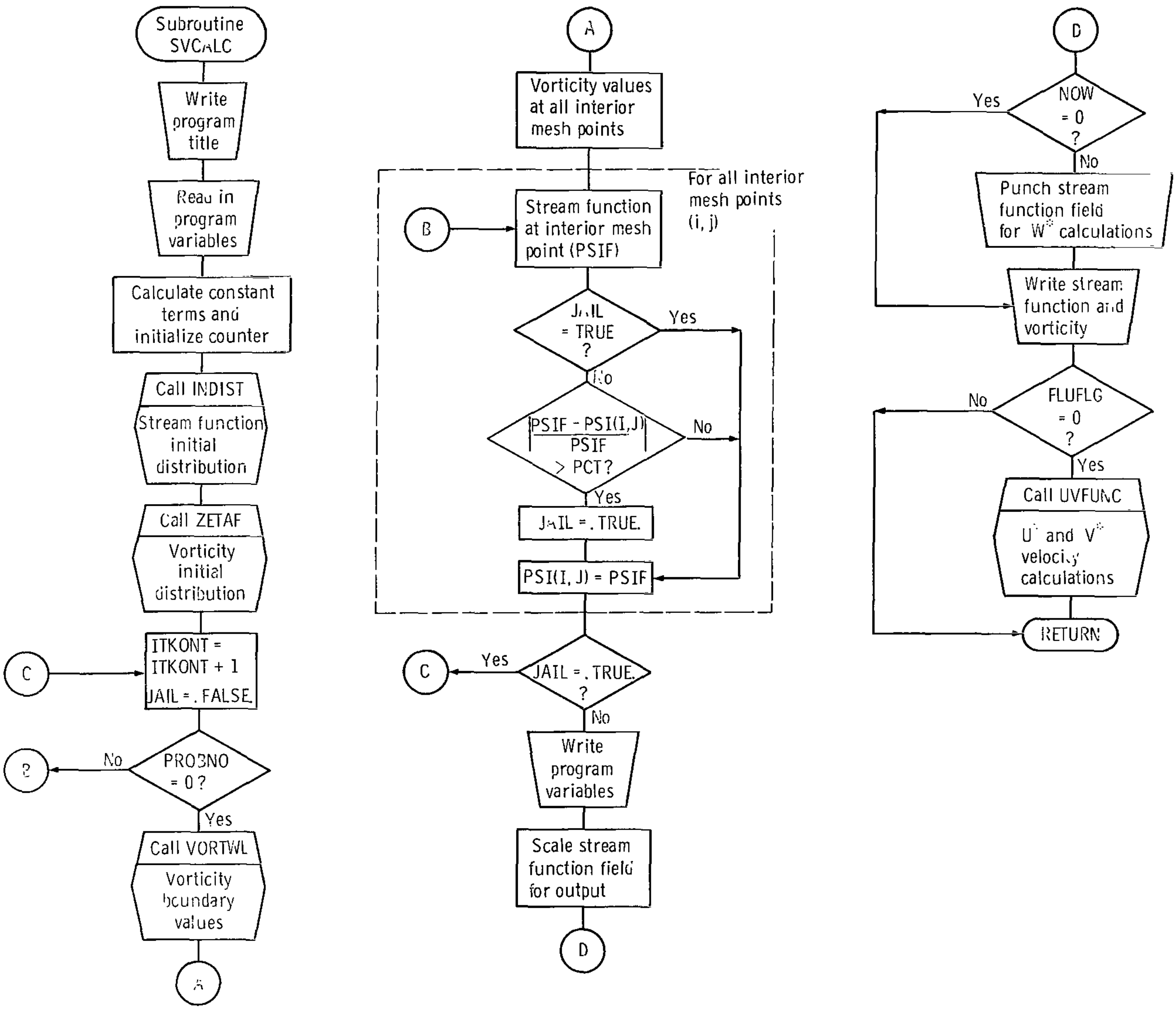

Figure 5. - Stream function and vorticity calculations.

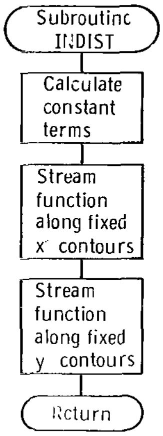

Figure 6. - Stream function initial distribution.

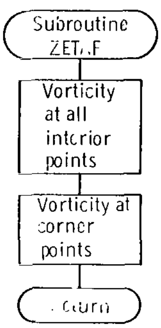

Figure 7. - Vorticity initial distribution.

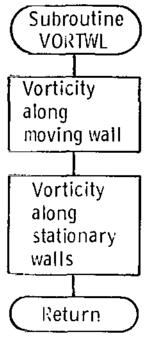

Figure 8. - Vorticity boundary wall values.

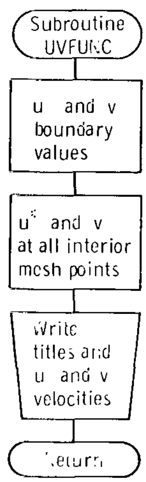

Figure 9. - u and v velocity profiles.

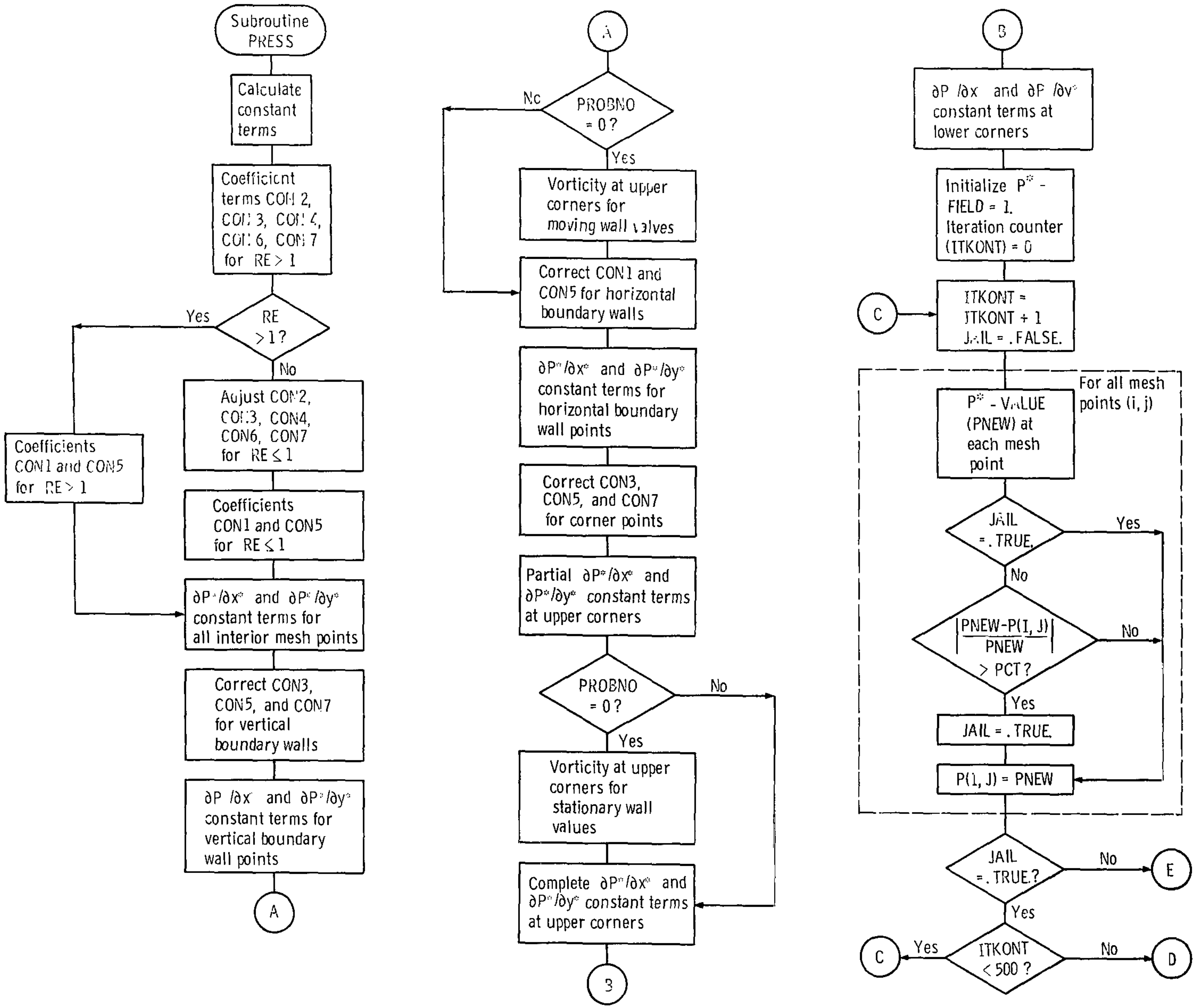

Figure 10. - Pressure field.

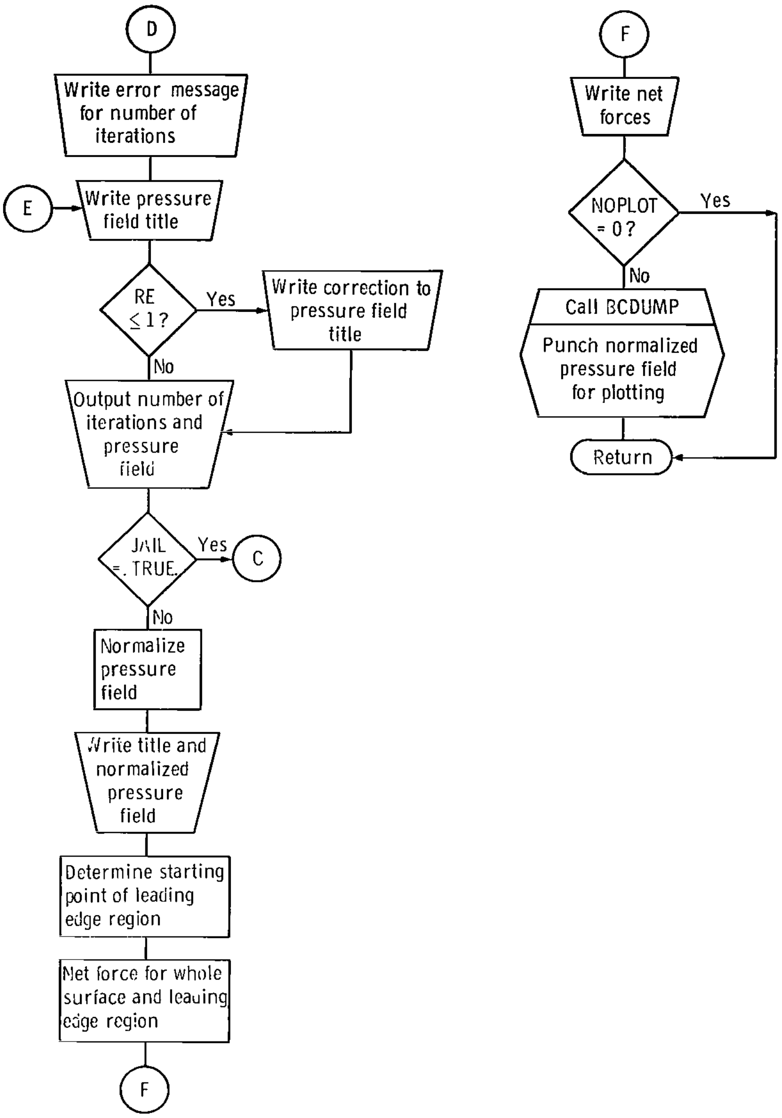

Figure 10. - Concluded.

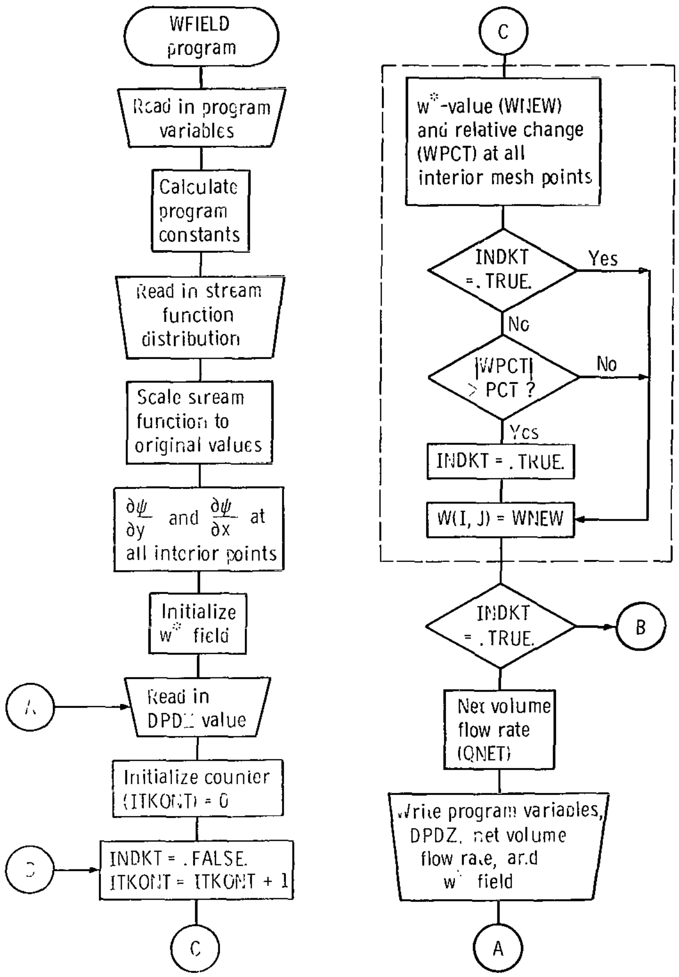

Figure 11. - w^*-velocity profile and Q^*_{inlet} calculation.

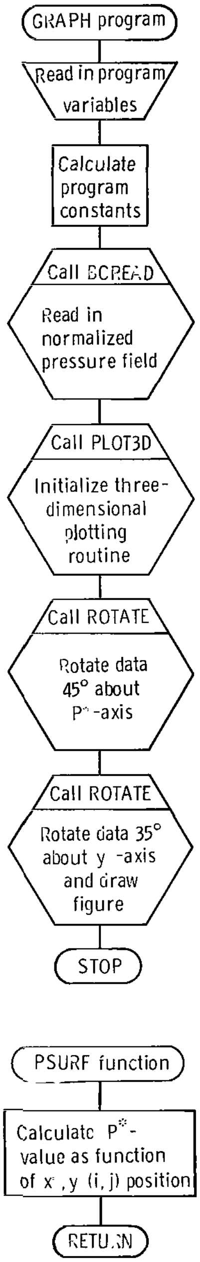

Figure 12. - Three-dimension pressure plot.

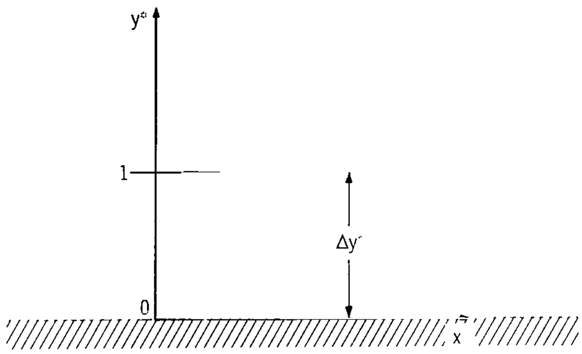

Figure 13. - Finite difference approximation of the boundary points.

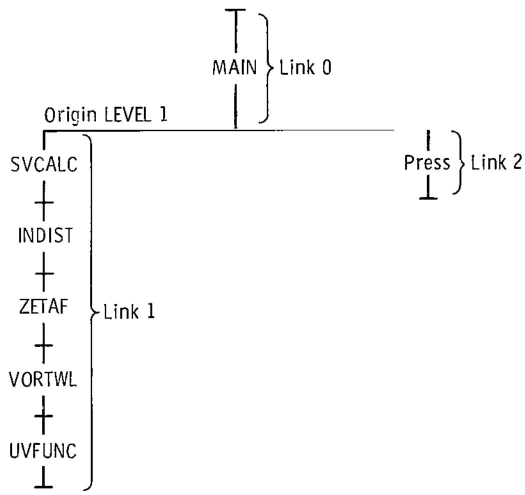

Figure 14. - Overlay structure of spiral grooved pumping seal model programs.

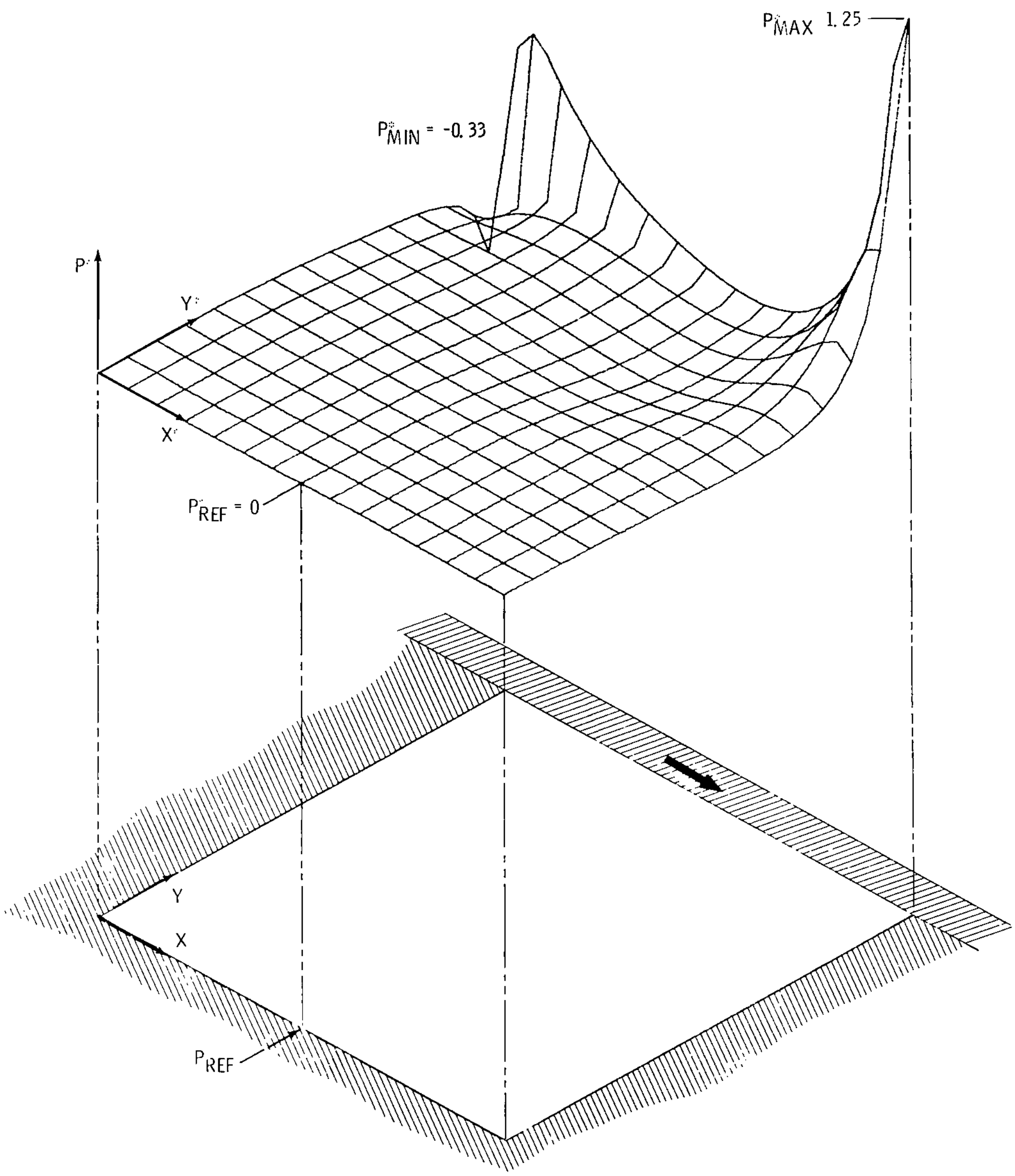

Figure 15. - Calcomp plot of normalized static pressure distribution due to cross flow in a square groove. Reynolds number, = 100.

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